Inverse Hyperbolic Function Integration Problem 1

$$\int \frac{\text{ArcTanh}[x]}{a+b\,x}\,dx$$

- *Rubi* integrates the expression by expanding it using the identity arctanh(z) = 1/2 (log(1+z)-log(1-z)):

```
Int[ArcTanh[x]/(a + b x), x]
```

$$\frac{\text{Log}[1+x]\,\text{Log}\left[\frac{a+b\,x}{a-b}\right]}{2\,b}-\frac{\text{Log}[1-x]\,\text{Log}\left[\frac{a+b\,x}{a+b}\right]}{2\,b}-\frac{\text{PolyLog}\left[2,\frac{b\,(1-x)}{a+b}\right]}{2\,b}+\frac{\text{PolyLog}\left[2,-\frac{b\,(1+x)}{a-b}\right]}{2\,b}$$

- *Mathematica* does *not* integrate the expression using the identity since it returns a complicated result:

$$\int \frac{\text{ArcTanh}[x]}{a+b\,x}\,dx$$

$$\frac{1}{8\,b}\Bigg(-\pi^2+4\,\text{ArcTanh}\left[\frac{a}{b}\right]^2+4\,i\,\pi\,\text{ArcTanh}[x]+8\,\text{ArcTanh}\left[\frac{a}{b}\right]\text{ArcTanh}[x]+8\,\text{ArcTanh}[x]^2-$$
$$4\,i\,\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[x]}\right]-8\,\text{ArcTanh}[x]\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[x]}\right]+8\,\text{ArcTanh}\left[\frac{a}{b}\right]\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right)}\right]+$$
$$8\,\text{ArcTanh}[x]\,\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right)}\right]+4\,i\,\pi\,\text{Log}\left[\frac{2}{\sqrt{1-x^2}}\right]+8\,\text{ArcTanh}[x]\,\text{Log}\left[\frac{2}{\sqrt{1-x^2}}\right]+$$
$$4\,\text{ArcTanh}[x]\,\text{Log}\left[1-x^2\right]+8\,\text{ArcTanh}[x]\,\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right]\right]-$$
$$8\,\text{ArcTanh}\left[\frac{a}{b}\right]\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right]\right]-$$
$$8\,\text{ArcTanh}[x]\,\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right]\right]-$$
$$4\,\text{PolyLog}\left[2,-e^{2\,\text{ArcTanh}[x]}\right]-4\,\text{PolyLog}\left[2,e^{-2\left(\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right)}\right]\Bigg)$$

$$\int \frac{\text{Log}[1+x]-\text{Log}[1-x]}{2\,(a+b\,x)}\,dx$$

$$\frac{\text{Log}[1+x]\,\text{Log}\left[\frac{a+b\,x}{a-b}\right]}{2\,b}-\frac{\text{Log}[1-x]\,\text{Log}\left[\frac{a+b\,x}{a+b}\right]}{2\,b}+\frac{\text{PolyLog}\left[2,\frac{b\,(1+x)}{-a+b}\right]}{2\,b}-\frac{\text{PolyLog}\left[2,\frac{b-b\,x}{a+b}\right]}{2\,b}$$

- *Maple* integrates the expression apparently by expanding it using the identity:

```
int(arctanh(x)/(a+b*x),x);
```

$$\frac{\text{Log}[1+x]\,\text{Log}\left[\frac{a+b\,x}{a-b}\right]}{2\,b}-\frac{\text{Log}[1-x]\,\text{Log}\left[\frac{a+b\,x}{a+b}\right]}{2\,b}-\frac{\text{PolyLog}\left[2,\frac{b\,(1-x)}{a+b}\right]}{2\,b}+\frac{\text{PolyLog}\left[2,-\frac{b\,(1+x)}{a-b}\right]}{2\,b}$$

Note that these systems give similar results to the above for the hyperbolic arccotangent function.

Inverse Hyperbolic Function Integration Problem 2

$$\int \frac{\text{ArcTanh}[x]}{a+b\,x+c\,x^2}\,dx$$

- *Rubi* is able to integrate the expression by expanding it using partial fraction expansion and the identity arctanh(z) = 1/2 (log(1+z)-log(1-z)):

```
Int[ArcTanh[x]/(a + b x + c x^2), x]
```

$$\frac{\text{Log}[1+x]\,\text{Log}\left[\frac{b-\sqrt{b^2-4ac}+2cx}{b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-\frac{\text{Log}[1-x]\,\text{Log}\left[-\frac{b-\sqrt{b^2-4ac}+2cx}{-b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}+$$
$$\frac{\text{Log}[1-x]\,\text{Log}\left[-\frac{b+\sqrt{b^2-4ac}+2cx}{-b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-\frac{\text{Log}[1+x]\,\text{Log}\left[\frac{b+\sqrt{b^2-4ac}+2cx}{b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left[2,-\frac{2c(1-x)}{-b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-$$
$$\frac{\text{PolyLog}\left[2,-\frac{2c(1-x)}{-b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left[2,-\frac{2c(1+x)}{b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-\frac{\text{PolyLog}\left[2,-\frac{2c(1+x)}{b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}$$

- *Mathematica* is unable to integrate the expression since it does *not* automatically expand it:

$$\int \frac{\text{ArcTanh}[x]}{a+b\,x+c\,x^2}\,dx$$

$$\int \frac{\text{ArcTanh}[x]}{a+b\,x+c\,x^2}\,dx$$

$$\int \frac{\text{Log}[1+x]-\text{Log}[1-x]}{2\left(a+b\,x+c\,x^2\right)}\,dx$$

$$-\frac{\text{Log}\left[1+\frac{2c(-1+x)}{b+2c-\sqrt{b^2-4ac}}\right]\text{Log}[1-x]}{2\sqrt{b^2-4ac}}+\frac{\text{Log}\left[1+\frac{2c(-1+x)}{b+2c+\sqrt{b^2-4ac}}\right]\text{Log}[1-x]}{2\sqrt{b^2-4ac}}-$$
$$\frac{\text{Log}[1+x]\,\text{Log}\left[1+\frac{2c(1+x)}{b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}+\frac{\text{Log}[1+x]\,\text{Log}\left[1-\frac{2c(1+x)}{-b+2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-\frac{\text{PolyLog}\left[2,-\frac{2c(-1+x)}{b+2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}+$$
$$\frac{\text{PolyLog}\left[2,-\frac{2c(-1+x)}{b+2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-\frac{\text{PolyLog}\left[2,-\frac{2c(1+x)}{b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left[2,\frac{2c(1+x)}{-b+2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}$$

$$\int \frac{2c\,\text{ArcTanh}[x]}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)}\,dx-\int \frac{2c\,\text{ArcTanh}[x]}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\,dx$$

$$
\begin{aligned}
&\frac{1}{2\sqrt{b^2-4ac}}\Bigg(\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]^2-\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]^2+2\,\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]\text{ArcTanh}[x]-\\
&2\,\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]\text{ArcTanh}[x]+2\,\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right)}\right]+\\
&2\,\text{ArcTanh}[x]\,\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right)}\right]-2\,\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]\\
&\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right)}\right]-2\,\text{ArcTanh}[x]\,\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right)}\right]+\\
&2\,\text{ArcTanh}[x]\,\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right]\right]-\\
&2\,\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right]\right]-\\
&2\,\text{ArcTanh}[x]\,\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right]\right]-\\
&2\,\text{ArcTanh}[x]\,\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right]\right]+\\
&2\,\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right]\right]+\\
&2\,\text{ArcTanh}[x]\,\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right]\right]-\\
&\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{b-\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right)}\right]+\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{b+\sqrt{b^2-4ac}}{2c}\right]+\text{ArcTanh}[x]\right)}\right]\Bigg)
\end{aligned}
$$

- *Maple* is able to integrate the expression:

```
int (arctanh (x) / (a + b * x + c * x^2), x);
```

$$
\begin{aligned}
&\frac{\text{Log}[1+x]\,\text{Log}\left[\frac{b-\sqrt{b^2-4ac}+2cx}{b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-\frac{\text{Log}[1-x]\,\text{Log}\left[-\frac{b-\sqrt{b^2-4ac}+2cx}{-b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}+\\
&\frac{\text{Log}[1-x]\,\text{Log}\left[-\frac{b+\sqrt{b^2-4ac}+2cx}{-b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-\frac{\text{Log}[1+x]\,\text{Log}\left[\frac{b+\sqrt{b^2-4ac}+2cx}{b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left[2,-\frac{2c\,(1-x)}{-b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-\\
&\frac{\text{PolyLog}\left[2,-\frac{2c\,(1-x)}{-b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left[2,-\frac{2c\,(1+x)}{b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}-\frac{\text{PolyLog}\left[2,-\frac{2c\,(1+x)}{b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}}
\end{aligned}
$$

Note that these systems give similar results to the above for the hyperbolic arccotangent function.

Inverse Hyperbolic Function Integration Problem 3

$$\int \frac{\text{ArcTanh}[a\, x^n]}{x}\, dx$$

- *Rubi* returns the polylog form of the rule for all symbolic and numeric n:

$$\text{Int}\left[\frac{\text{ArcTanh}[a\, x^n]}{x},\ x\right]$$

$$-\frac{\text{PolyLog}[2,\ -a\, x^n]}{2\, n} + \frac{\text{PolyLog}[2,\ a\, x^n]}{2\, n}$$

$$\text{Int}\left[\frac{\text{ArcTanh}[a\, x^5]}{x},\ x\right]$$

$$-\frac{1}{10}\,\text{PolyLog}[2,\ -a\, x^5] + \frac{1}{10}\,\text{PolyLog}[2,\ a\, x^5]$$

- *Mathematica* returns the hypergeometric form of the rule for symbolic n, but the polylog form for numeric n:

$$\int \frac{\text{ArcTanh}[a\, x^n]}{x}\, dx$$

$$\frac{a\, x^n\, \text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\ \frac{1}{2},\ 1\right\},\ \left\{\frac{3}{2},\ \frac{3}{2}\right\},\ a^2\, x^{2\, n}\right]}{n}$$

$$\int \frac{\text{ArcTanh}[a\, x^5]}{x}\, dx$$

$$-\frac{1}{10}\,\text{PolyLog}[2,\ -a\, x^5] + \frac{1}{10}\,\text{PolyLog}[2,\ a\, x^5]$$

- *Maple* returns the polylog form of the rule for symbolic n, but an expression not in closed-form when n is an integer:

```
int(arctanh(a*x^n)/x, x);
```

$$\frac{\text{dilog}(1 - a\, x^n)}{2\, n} - \frac{\text{dilog}(1 + a\, x^n)}{2\, n}$$

```
int(arctanh(a*x^5)/x, x);
```

```
1/2*ln(x)*ln(1+a*x^5) -
 5/2*a*sum(1/5/a*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1)), _R1 = RootOf(_Z^5*a+1)) -
 1/2*ln(x)*ln(1-a*x^5) +
 5/2*a*sum(1/5/a*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1)), _R1 = RootOf(_Z^5*a-1))
```

Note that these systems give similar results to the above for the hyperbolic arccotangent function.

Inverse Hyperbolic Function Integration Problem 4

$$\int \text{ArcTanh}[a\,\text{Tanh}[x]]\,dx$$

- *Rubi* returns a 5 term sum free of the imaginary unit:

```
Int[ArcTanh[a Tanh[x]], x]
```

$$x\,\text{ArcTanh}[a\,\text{Tanh}[x]] - \frac{1}{2}\,x\,\text{Log}\left[1 + \frac{\left(1-a^2\right) e^{2x}}{1-2a+a^2}\right] + \frac{1}{2}\,x\,\text{Log}\left[1 + \frac{\left(1-a^2\right) e^{2x}}{1+2a+a^2}\right] - \frac{1}{4}\,\text{PolyLog}\left[2, -\frac{\left(1-a^2\right) e^{2x}}{1-2a+a^2}\right] + \frac{1}{4}\,\text{PolyLog}\left[2, -\frac{\left(1-a^2\right) e^{2x}}{1+2a+a^2}\right]$$

- *Mathematica* returns a large complicated expression involving the imaginary unit:

```
∫ArcTanh[a Tanh[x]] dx
```

$$x\,\text{ArcTanh}[a\,\text{Tanh}[x]] - \frac{1}{4\sqrt{-a^2}}\,a \left(-4\,x\,\text{ArcTan}\left[\frac{\text{Coth}[x]}{\sqrt{-a^2}}\right] + 2\,i\,\text{ArcCos}\left[\frac{1+a^2}{-1+a^2}\right] \text{ArcTan}\left[\sqrt{-a^2}\,\text{Tanh}[x]\right] + \left(\text{ArcCos}\left[\frac{1+a^2}{-1+a^2}\right] - 2\left(\text{ArcTan}\left[\frac{\text{Coth}[x]}{\sqrt{-a^2}}\right] + \text{ArcTan}\left[\sqrt{-a^2}\,\text{Tanh}[x]\right]\right)\right) \text{Log}\left[\frac{\sqrt{2}\,\sqrt{-a^2}\,e^{-x}}{\sqrt{-1+a^2}\,\sqrt{-1-a^2+\left(-1+a^2\right)\text{Cosh}[2x]}}\right] + \left(\text{ArcCos}\left[\frac{1+a^2}{-1+a^2}\right] + 2\left(\text{ArcTan}\left[\frac{\text{Coth}[x]}{\sqrt{-a^2}}\right] + \text{ArcTan}\left[\sqrt{-a^2}\,\text{Tanh}[x]\right]\right)\right) \text{Log}\left[\frac{\sqrt{2}\,\sqrt{-a^2}\,e^{x}}{\sqrt{-1+a^2}\,\sqrt{-1-a^2+\left(-1+a^2\right)\text{Cosh}[2x]}}\right] - \left(\text{ArcCos}\left[\frac{1+a^2}{-1+a^2}\right] - 2\,\text{ArcTan}\left[\sqrt{-a^2}\,\text{Tanh}[x]\right]\right) \left(\text{Log}[2] + \text{Log}\left[-\frac{\left(i\,a^2+\sqrt{-a^2}\right)\left(-1+\text{Tanh}[x]\right)}{\left(-1+a^2\right)\left(i+\sqrt{-a^2}\,\text{Tanh}[x]\right)}\right]\right) - \left(\text{ArcCos}\left[\frac{1+a^2}{-1+a^2}\right] + 2\,\text{ArcTan}\left[\sqrt{-a^2}\,\text{Tanh}[x]\right]\right) \left(\text{Log}[2] + \text{Log}\left[-\frac{\left(-i\,a^2+\sqrt{-a^2}\right)\left(1+\text{Tanh}[x]\right)}{\left(-1+a^2\right)\left(i+\sqrt{-a^2}\,\text{Tanh}[x]\right)}\right]\right) + i\left(-\text{PolyLog}\left[2, \frac{\left(1+a^2-2\,i\,\sqrt{-a^2}\right)\left(-i+\sqrt{-a^2}\,\text{Tanh}[x]\right)}{\left(-1+a^2\right)\left(i+\sqrt{-a^2}\,\text{Tanh}[x]\right)}\right] + \text{PolyLog}\left[2, \frac{\left(1+a^2+2\,i\,\sqrt{-a^2}\right)\left(-i+\sqrt{-a^2}\,\text{Tanh}[x]\right)}{\left(-1+a^2\right)\left(i+\sqrt{-a^2}\,\text{Tanh}[x]\right)}\right]\right)\right)$$

Maple returns a large complicated expression involving the imaginary unit:

```
int(arctanh(a*tanh(x)), x);
```

```
-1/2*a*x/(1+a)*ln((((1+a)*(a-1))^(1/2)-exp(x)-a*exp(x))/((1+a)*(a-1))^(1/2))-
 1/2*a*x/(1+a)*ln((((1+a)*(a-1))^(1/2)+exp(x)+a*exp(x))/((1+a)*(a-1))^(1/2))+
 1/2*a/(a-1)*ln(exp(x))*ln
  ((((1+a)*(a-1))^(1/2)+exp(x)-a*exp(x))/((1+a)*(a-1))^(1/2))+1/2*a/(a-1)*
  ln(exp(x))*ln((((1+a)*(a-1))^(1/2)-exp(x)+a*exp(x))/((1+a)*(a-1))^(1/2))-
 1/2*I*Pi*x+1/2*I*Pi*csgn(I/(1+exp(2*x))*((-1+exp(2*x))*a-exp(2*x)-1))^2*x-
 1/4*I*Pi*csgn(I/(1+exp(2*x))*((-1+exp(2*x))*a+exp(2*x)+1))^3*x-
 1/4*I*Pi*csgn(I/(1+exp(2*x))*((-1+exp(2*x))*a-exp(2*x)-1))^3*x-
 1/4*I*Pi*csgn(I/(1+exp(2*x))*((-1+exp(2*x))*a-exp(2*x)-1))^2*
  csgn(I*((-1+exp(2*x))*a-exp(2*x)-1))*x+1/4*I*Pi*csgn
  (I/(1+exp(2*x))*((-1+exp(2*x))*a+exp(2*x)+1))^2*csgn(I/(1+exp(2*x)))*x+
 1/4*I*Pi*csgn(I/(1+exp(2*x))*((-1+exp(2*x))*a+exp(2*x)+1))^2*
  csgn(I*((-1+exp(2*x))*a+exp(2*x)+1))*x+
 1/4*I*Pi*csgn(I/(1+exp(2*x))*((-1+exp(2*x))*a-exp(2*x)-1))*csgn
  (I/(1+exp(2*x)))*csgn(I*((-1+exp(2*x))*a-exp(2*x)-1))*x-1/4*I*Pi*csgn
  (I/(1+exp(2*x))*((-1+exp(2*x))*a-exp(2*x)-1))^2*csgn(I/(1+exp(2*x)))*x-
 1/4*I*Pi*csgn(I/(1+exp(2*x))*((-1+exp(2*x))*a+exp(2*x)+1))*csgn
  (I/(1+exp(2*x)))*csgn(I*((-1+exp(2*x))*a+exp(2*x)+1))*x+
 1/2*a/(a-1)*dilog((((1+a)*(a-1))^(1/2)-exp(x)+a*exp(x))/((1+a)*(a-1))^(1/2))+
 1/2*a/(a-1)*dilog((((1+a)*(a-1))^(1/2)+exp(x)-a*exp(x))/((1+a)*(a-1))^(1/2))-
 1/2*a/(1+a)*dilog((((1+a)*(a-1))^(1/2)-exp(x)-a*exp(x))/((1+a)*(a-1))^(1/2))-
 1/2*a/(1+a)*dilog((((1+a)*(a-1))^(1/2)+exp(x)+a*exp(x))/((1+a)*(a-1))^(1/2))-
 1/2*x*(ln((((1+a)*(a-1))^(1/2)-exp(x)-a*exp(x))/((1+a)*(a-1))^(1/2))+
     ln((((1+a)*(a-1))^(1/2)+exp(x)+a*exp(x))/((1+a)*(a-1))^(1/2)))/(1+a)-
 1/2/(a-1)*ln(exp(x))*ln((((1+a)*(a-1))^(1/2)-exp(x)+a*exp(x))/
    ((1+a)*(a-1))^(1/2))-1/2/(a-1)*ln(exp(x))*ln
  ((((1+a)*(a-1))^(1/2)+exp(x)-a*exp(x))/((1+a)*(a-1))^(1/2))+
 1/2*x*ln((-1+exp(2*x))*a+exp(2*x)+1)-
 1/2*ln(exp(x))*ln(-a+a*exp(2*x)-1-exp(2*x))-
 1/2/(a-1)*dilog((((1+a)*(a-1))^(1/2)+exp(x)-a*exp(x))/((1+a)*(a-1))^(1/2))-
 1/2/(a-1)*dilog((((1+a)*(a-1))^(1/2)-exp(x)+a*exp(x))/((1+a)*(a-1))^(1/2))-
 1/2/(1+a)*dilog((((1+a)*(a-1))^(1/2)-exp(x)-a*exp(x))/((1+a)*(a-1))^(1/2))-
 1/2/(1+a)*dilog((((1+a)*(a-1))^(1/2)+exp(x)+a*exp(x))/((1+a)*(a-1))^(1/2))
```

Note that these systems give similar results to the above for the hyperbolic arccotangent function.

Inverse Hyperbolic Function Integration Problem 5

$$\int \text{ArcSinh}\left[e^{a+b\,x}\right] dx$$

- *Rubi* uses the substitution u=a+b x to generalize rule:

```
Int[ArcSinh[E^x], x]
```

$$\frac{1}{2}\text{ArcSinh}[e^x]^2+\text{ArcSinh}[e^x]\,\text{Log}\left[1-e^{-2\,\text{ArcSinh}[e^x]}\right]-\frac{1}{2}\text{PolyLog}\left[2,\,e^{-2\,\text{ArcSinh}[e^x]}\right]$$

```
Int[ArcSinh[E^(a+b x)], x]
```

$$\frac{\text{ArcSinh}\left[e^{a+b\,x}\right]^2}{2\,b}+\frac{\text{ArcSinh}\left[e^{a+b\,x}\right]\text{Log}\left[1-e^{-2\,\text{ArcSinh}\left[e^{a+b\,x}\right]}\right]}{b}-\frac{\text{PolyLog}\left[2,\,e^{-2\,\text{ArcSinh}\left[e^{a+b\,x}\right]}\right]}{2\,b}$$

- *Mathematica* does not use the substitution u=a+b x to generalize rule:

$$\int \text{ArcSinh}[e^x]\, dx$$

$$x\,\text{ArcSinh}[e^x]+\frac{1}{2}\text{ArcSinh}[e^x]^2+\text{ArcSinh}[e^x]\,\text{Log}\left[1-e^{-2\,\text{ArcSinh}[e^x]}\right]-x\,\text{Log}\left[e^x+\sqrt{1+e^{2\,x}}\right]-\frac{1}{2}\text{PolyLog}\left[2,\,e^{-2\,\text{ArcSinh}[e^x]}\right]$$

$$\int \text{ArcSinh}\left[e^{a+b\,x}\right] dx$$

$$\int \text{ArcSinh}\left[e^{a+b\,x}\right] dx$$

- *Maple* is unable to integrate either expression:

```
int(arcsinh(exp(x)), x);
```

$$\int \text{ArcSinh}[e^x]\, dx$$

```
int(arcsinh(exp(a+b*x)), x);
```

$$\int \text{ArcSinh}\left[e^{a+b\,x}\right] dx$$

Note that these systems give similar results to the above for the hyperbolic arccosine function.