Exponential Function Integration Problem 1

$$\int x^m e^{(a+b x)^3} \, dx$$

- *Rubi* is able to integrate $x^m e^{(a+b x)^3}$ for integer $m \geq 0$:

```
Int[E^(a+b x)^3, x]
```

$$-\frac{(a+b x)\,\text{Gamma}\left[\frac{1}{3},-(a+b x)^3\right]}{3 b \left(-(a+b x)^3\right)^{1/3}}$$

```
Int[x E^(a+b x)^3, x]
```

$$\frac{a (a+b x)\,\text{Gamma}\left[\frac{1}{3},-(a+b x)^3\right]}{3 b^2 \left(-(a+b x)^3\right)^{1/3}}-\frac{(a+b x)^2\,\text{Gamma}\left[\frac{2}{3},-(a+b x)^3\right]}{3 b^2 \left(-(a+b x)^3\right)^{2/3}}$$

```
Int[x^2 E^(a+b x)^3, x]
```

$$\frac{e^{(a+b x)^3}}{3 b^3}-\frac{a^2 (a+b x)\,\text{Gamma}\left[\frac{1}{3},-(a+b x)^3\right]}{3 b^3 \left(-(a+b x)^3\right)^{1/3}}+\frac{2 a (a+b x)^2\,\text{Gamma}\left[\frac{2}{3},-(a+b x)^3\right]}{3 b^3 \left(-(a+b x)^3\right)^{2/3}}$$

- *Mathematica* is only able to integrate $x^m e^{(a+b x)^3}$ for $m = 0$:

$$\int e^{(a+b x)^3} \, dx$$

$$-\frac{(a+b x)\,\text{Gamma}\left[\frac{1}{3},-(a+b x)^3\right]}{3 b \left(-(a+b x)^3\right)^{1/3}}$$

$$\int x\, e^{(a+b x)^3} \, dx$$

$$\int e^{(a+b x)^3} x \, dx$$

$$\int x^2 e^{(a+b x)^3} \, dx$$

$$\int e^{(a+b x)^3} x^2 \, dx$$

- *Maple* is unable to integrate $x^m e^{(a+b x)^3}$ for any integer m:

```
int(exp((a+b*x)^3), x);
```

$$\int e^{(a+b x)^3} \, dx$$

```
int (x * exp ((a + b * x) ^3), x);
```

$$\int x\, e^{(a+b\,x)^3}\, dx$$

```
int (x^2 * exp ((a + b * x) ^3), x);
```

$$\int x^2\, e^{(a+b\,x)^3}\, dx$$

Exponential Function Integration Problem 2

$$\int x^m\, e^{(a+b\,x)^n}\, dx$$

- *Rubi* is able to integrate $x^m\, e^{(a+b\,x)^n}$ for integer $m \geq 0$:

```
Int[e^((a+b x)^n), x]
```

$$-\frac{(a+b\,x)\,\left(-(a+b\,x)^n\right)^{-1/n}\,\mathrm{Gamma}\left[\frac{1}{n},\,-(a+b\,x)^n\right]}{b\,n}$$

```
Int[x e^((a+b x)^n), x]
```

$$\frac{a\,(a+b\,x)\,\left(-(a+b\,x)^n\right)^{-1/n}\,\mathrm{Gamma}\left[\frac{1}{n},\,-(a+b\,x)^n\right]}{b^2\,n}-\frac{(a+b\,x)^2\,\left(-(a+b\,x)^n\right)^{-2/n}\,\mathrm{Gamma}\left[\frac{2}{n},\,-(a+b\,x)^n\right]}{b^2\,n}$$

```
Int[x^2 e^((a+b x)^n), x]
```

$$-\frac{a^2\,(a+b\,x)\,\left(-(a+b\,x)^n\right)^{-1/n}\,\mathrm{Gamma}\left[\frac{1}{n},\,-(a+b\,x)^n\right]}{b^3\,n}+\frac{2\,a\,(a+b\,x)^2\,\left(-(a+b\,x)^n\right)^{-2/n}\,\mathrm{Gamma}\left[\frac{2}{n},\,-(a+b\,x)^n\right]}{b^3\,n}-\frac{(a+b\,x)^3\,\left(-(a+b\,x)^n\right)^{-3/n}\,\mathrm{Gamma}\left[\frac{3}{n},\,-(a+b\,x)^n\right]}{b^3\,n}$$

- *Mathematica* is only able to integrate $x^m\, e^{(a+b\,x)^n}$ for $m = 0$:

$$\int e^{(a+b\,x)^n}\, dx$$

$$-\frac{(a+b\,x)\,\left(-(a+b\,x)^n\right)^{-1/n}\,\mathrm{Gamma}\left[\frac{1}{n},\,-(a+b\,x)^n\right]}{b\,n}$$

$$\int x\, e^{(a+b\,x)^n}\, dx$$

$$\int e^{(a+b\,x)^n}\, x\, dx$$

$$\int x^2\, e^{(a+b\,x)^n}\, dx$$

$$\int e^{(a+b\,x)^n}\, x^2\, dx$$

- *Maple* is unable to integrate $x^m\, e^{(a+b\,x)^n}$ for any integer m:

```
int(exp((a+b*x)^n), x);
```

$$\int e^{(a+b\,x)^n}\, dx$$

```
int (x * exp ((a + b * x)^n), x);
```

$$\int x\, e^{(a+b\,x)^n}\, dx$$

```
int (x^2 * exp ((a + b * x)^n), x);
```

$$\int x^2\, e^{(a+b\,x)^n}\, dx$$

Exponential Function Integration Problem 3

$$\int (a+b\,x)^m\, e^{(a+b\,x)^n}\, dx$$

- *Rubi* is able to integrate the generalized expression using the trivial substitution $u = a + b\,x$:

```
Int[x^m e^(x^n), x]
```

$$-\frac{x^{1+m}\left(-x^n\right)^{-\frac{1+m}{n}}\,\text{Gamma}\left[\frac{1+m}{n},\,-x^n\right]}{n}$$

```
Int[(a + b x)^m e^((a+b x)^n), x]
```

$$-\frac{(a+b\,x)^{1+m}\left(-(a+b\,x)^n\right)^{-\frac{1+m}{n}}\,\text{Gamma}\left[\frac{1+m}{n},\,-(a+b\,x)^n\right]}{b\,n}$$

- *Mathematica* is unable to integrate the generalized expression:

$$\int x^m\, e^{x^n}\, dx$$

$$-\frac{x^{1+m}\left(-x^n\right)^{-\frac{1+m}{n}}\,\text{Gamma}\left[\frac{1+m}{n},\,-x^n\right]}{n}$$

$$\int (a+b\,x)^m\, e^{(a+b\,x)^n}\, dx$$

$$\int (a+b\,x)^m\, e^{(a+b\,x)^n}\, dx$$

- *Maple* is unable to integrate either expression:

```
int(x^m*exp(x^n), x);
```

$$\int x^m\, e^{x^n}\, dx$$

```
int((a+b*x)^m*exp((a+b*x)^n), x);
```

$$\int (a+b\,x)^m\, e^{(a+b\,x)^n}\, dx$$

Exponential Function Integration Problem 4

$$\int \frac{8^x}{a + b\,4^x}\, dx$$

- The *Rubi* result involves only elementary functions:

```
Int[8^x/(a + b 4^x), x]
```

$$\frac{2^x}{b\,\mathrm{Log}[2]} - \frac{\sqrt{a}\,\mathrm{ArcTan}\left[\frac{2^x\sqrt{b}}{\sqrt{a}}\right]}{b^{3/2}\,\mathrm{Log}[2]}$$

- The *Mathematica* result involves nonelementary functions:

$$\int \frac{8^x}{a + b\,4^x}\, dx$$

$$\frac{8^x\,\mathrm{Hypergeometric2F1}\left[1, \frac{\mathrm{Log}[8]}{\mathrm{Log}[4]}, \frac{\mathrm{Log}[32]}{\mathrm{Log}[4]}, -\frac{4^x\,b}{a}\right]}{a\,\mathrm{Log}[8]}$$

- The *Maple* result involves only elementary functions, but is more complicated:

```
int(8^x/(a+b*4^x),x);
```

$$\frac{2^x}{b\,\mathrm{Log}[2]} + \frac{\sqrt{-a\,b}\,\mathrm{Log}\left[2^x - \frac{\sqrt{-a\,b}}{b}\right]}{2\,b^2\,\mathrm{Log}[2]} - \frac{\sqrt{-a\,b}\,\mathrm{Log}\left[2^x + \frac{\sqrt{-a\,b}}{b}\right]}{2\,b^2\,\mathrm{Log}[2]}$$

Exponential Function Integration Problem 5

$$\int \frac{4^x}{(a + b\, 2^x)^m}\, dx$$

- Both *Rubi* results are correct:

```
Int[4^x/(a + b 2^x), x]
```

$$\frac{2^x}{b\, \mathrm{Log}[2]} - \frac{a\, \mathrm{Log}[a + 2^x\, b]}{b^2\, \mathrm{Log}[2]}$$

```
Int[4^x/Sqrt[a + b 2^x], x]
```

$$-\frac{4\, a\, \sqrt{a + 2^x\, b}}{3\, b^2\, \mathrm{Log}[2]} + \frac{2^{1+x}\, \sqrt{a + 2^x\, b}}{3\, b\, \mathrm{Log}[2]}$$

- Both *Mathematica* results are incorrect:

$$\int \frac{4^x}{a + b\, 2^x}\, dx$$

$$\frac{\mathrm{Log}[-a - 2^x\, b]}{b\, \mathrm{Log}[4]}$$

$$\int \frac{4^x}{\mathrm{Sqrt}[a + b\, 2^x]}\, dx$$

$$\frac{2\, \sqrt{a + 2^x\, b}}{b\, \mathrm{Log}[4]}$$

- The second *Maple* result is incorrect:

```
int(4^x/(a+b*2^x),x);
```

$$\frac{2^x\, b - a\, \mathrm{Log}[a + 2^x\, b]}{b^2\, \mathrm{Log}[2]}$$

```
int(4^x/sqrt(a+b*2^x),x);
```

$$-\frac{2\, (2\, a - b\, 2^x)\, \sqrt{a + b\, 2^x}}{\left(2 + b^2\right)\, \mathrm{Log}[2]}$$

Exponential Function Integration Problem 6

$$\int e^{a+b x^n} e^{c+d x^n} \, dx$$

- *Rubi* is able to integrate the expression for numeric and symbolic n:

```
Int[e^(a+b x^n) e^(c+d x^n), x]
```

$$-\frac{e^{a+c} x \left(-(b+d) x^n\right)^{-1/n} \text{Gamma}\left[\frac{1}{n}, -(b+d) x^n\right]}{n}$$

```
Int[e^(a+b x^5) e^(c+d x^5), x]
```

$$-\frac{e^{a+c} x \, \text{Gamma}\left[\frac{1}{5}, -(b+d) x^5\right]}{5 \left(-(b+d) x^5\right)^{1/5}}$$

```
Int[e^(a+b x^(2/3)) e^(c+d x^(2/3)), x]
```

$$\frac{3 e^{a+c+(b+d) x^{2/3}} x^{1/3}}{2 (b+d)} - \frac{3 e^{a+c} \sqrt{\pi} \, \text{Erfi}\left[\sqrt{b+d} \, x^{1/3}\right]}{4 (b+d)^{3/2}}$$

- *Mathematica* is unable to integrate the expression unless n is numeric:

$$\int e^{a+b x^n} e^{c+d x^n} \, dx$$

$$\int e^{a+c+b x^n+d x^n} \, dx$$

$$\int e^{a+b x^5} e^{c+d x^5} \, dx$$

$$-\frac{e^{a+c} x \, \text{Gamma}\left[\frac{1}{5}, -(b+d) x^5\right]}{5 \left(-(b+d) x^5\right)^{1/5}}$$

$$\int e^{a+b x^{2/3}} e^{c+d x^{2/3}} \, dx$$

$$\frac{3 e^{a+c+(b+d) x^{2/3}} x^{1/3}}{2 (b+d)} - \frac{3 e^{a+c} \sqrt{\pi} \, \text{Erfi}\left[\sqrt{b+d} \, x^{1/3}\right]}{4 (b+d)^{3/2}}$$

- *Maple* is unable to integrate the expression unless n is numeric:

```
int(exp(a+b*x^n)*exp(c+d*x^n), x);
```

$$\int e^{a+b x^n} e^{c+d x^n} \, dx$$

```
int(exp(a+b*x^5)*exp(c+d*x^5), x);
```

$$\frac{\text{Exp}[a+c]\, x\left(-\text{Gamma}\left[\frac{1}{5}, -(b+d)\, x^5\right] \text{Gamma}\left[\frac{4}{5}\right] \text{Sin}\left[\frac{\pi}{5}\right] + \pi\right)}{5\left(\text{Sin}\left[\frac{\pi}{5}\right] \left(-(b+d)\, x^5\right)^{1/5} \text{Gamma}\left[\frac{4}{5}\right]\right)}$$

```
int (exp (a + b * x^(2 / 3)) * exp (c + d * x^(2 / 3)), x);
```

$$\frac{3\, e^{a+c+(b+d)\, x^{2/3}}\, x^{1/3}}{2\,(b+d)} - \frac{3\, e^{a+c} \sqrt{\pi}\, \text{Erfi}\left[\sqrt{b+d}\; x^{1/3}\right]}{4\,(b+d)^{3/2}}$$