Integral Function Integration Problem 1

$$\int x^m \, \text{ExpIntegralEi}[a + b\,x]^2 \, dx$$

- *Rubi* is able integrate x^m **ExpIntegralEi[a + b x]^2** for integer $m \geq 0$:

```
Int[ExpIntegralEi[a + b x]^2, x]
```

$$-\frac{2\,e^{a+b\,x}\,\text{ExpIntegralEi}[a+b\,x]}{b}+\frac{(a+b\,x)\,\text{ExpIntegralEi}[a+b\,x]^2}{b}+\frac{2\,\text{ExpIntegralEi}[2\,(a+b\,x)]}{b}$$

```
Int[x ExpIntegralEi[a + b x]^2, x]
```

$$\frac{e^{2\,a+2\,b\,x}}{2\,b^2}+\frac{e^{a+b\,x}\,(1+a-b\,x)\,\text{ExpIntegralEi}[a+b\,x]}{b^2}-\frac{1}{2}\left(\frac{a^2}{b^2}-x^2\right)\text{ExpIntegralEi}[a+b\,x]^2-\frac{(1+2\,a)\,\text{ExpIntegralEi}[2\,(a+b\,x)]}{b^2}$$

```
Int[x^2 ExpIntegralEi[a + b x]^2, x]
```

$$-\frac{5\,e^{2\,a+2\,b\,x}}{6\,b^3}-\frac{2\,a\,e^{2\,a+2\,b\,x}}{3\,b^3}+\frac{e^{2\,a+2\,b\,x}\,x}{3\,b^2}-\frac{2\,e^{a+b\,x}\left(2+a+a^2-2\,b\,x-a\,b\,x+b^2\,x^2\right)\text{ExpIntegralEi}[a+b\,x]}{3\,b^3}+\frac{1}{3}\left(\frac{a^3}{b^3}+x^3\right)\text{ExpIntegralEi}[a+b\,x]^2+\frac{2\left(2+3\,a+3\,a^2\right)\text{ExpIntegralEi}[2\,(a+b\,x)]}{3\,b^3}$$

- *Mathematica* is unable integrate x^m **ExpIntegralEi[a + b x]^2** for integer $m > 0$:

$$\int \text{ExpIntegralEi}[a+b\,x]^2\,dx$$

$$\frac{-2\,e^{a+b\,x}\,\text{ExpIntegralEi}[a+b\,x]+(a+b\,x)\,\text{ExpIntegralEi}[a+b\,x]^2+2\,\text{ExpIntegralEi}[2\,(a+b\,x)]}{b}$$

$$\int x\,\text{ExpIntegralEi}[a+b\,x]^2\,dx$$

$$\int x\,\text{ExpIntegralEi}[a+b\,x]^2\,dx$$

$$\int x^2\,\text{ExpIntegralEi}[a+b\,x]^2\,dx$$

$$\int x^2\,\text{ExpIntegralEi}[a+b\,x]^2\,dx$$

- *Maple* is unable integrate x^m **ExpIntegralEi[a + b x]^2** for integer $m > 1$:

```
int(Ei(a+b*x)^2, x);
```

$$\frac{-2\, e^{a+b\,x}\, \text{Ei}[a+b\,x] + (a+b\,x)\, \text{Ei}[a+b\,x]^2 - 2\, \text{Ei}[1, -2\,a-2\,b\,x]}{b}$$

```
int (x * Ei (a + b * x) ^2, x);
```

$$\frac{1}{2\,b^2}\Big(e^{2\,a+2\,b\,x} + 2\,e^{a+b\,x}\,\text{Ei}[a+b\,x] + 2\,a\,e^{a+b\,x}\,\text{Ei}[a+b\,x] - 2\,b\,e^{a+b\,x}\,x\,\text{Ei}[a+b\,x] - a^2\,\text{Ei}[a+b\,x]^2 + b^2\,x^2\,\text{Ei}[a+b\,x]^2 - 2\,\text{Ei}[2\,(a+b\,x)] + 4\,a\,\text{Ei}[1, -2\,a-2\,b\,x]\Big)$$

```
int (x^2 * Ei (a + b * x) ^2, x);
```

$$\int x^2\,\text{Ei}[a+b\,x]^2\,dx$$

Integral Function Integration Problem 2

$$\int x^m \text{SinIntegral}[a + b x]^2 \, dx$$

- *Rubi* is able integrate x^m SinIntegral[a + b x]^2 for integer $m \geq 0$:

```
Int[SinIntegral[a + b x]^2, x]
```

$$\frac{2 \text{Cos}[a + b x] \text{SinIntegral}[a + b x]}{b} + \frac{(a + b x) \text{SinIntegral}[a + b x]^2}{b} - \frac{\text{SinIntegral}[2 (a + b x)]}{b}$$

```
Int[x SinIntegral[a + b x]^2, x]
```

$$\frac{\text{Cos}[2 a + 2 b x]}{4 b^2} - \frac{\text{CosIntegral}[2 (a + b x)]}{2 b^2} + \frac{\text{Log}[a + b x]}{2 b^2} - \frac{((a - b x) \text{Cos}[a + b x] + \text{Sin}[a + b x]) \text{SinIntegral}[a + b x]}{b^2} - \frac{1}{2} \left(\frac{a^2}{b^2} - x^2\right) \text{SinIntegral}[a + b x]^2 + \frac{a \text{SinIntegral}[2 (a + b x)]}{b^2}$$

```
Int[x^2 SinIntegral[a + b x]^2, x]
```

$$\frac{2 x}{3 b^2} - \frac{a \text{Cos}[2 a + 2 b x]}{3 b^3} + \frac{x \text{Cos}[2 a + 2 b x]}{6 b^2} + \frac{a \text{CosIntegral}[2 (a + b x)]}{b^3} - \frac{a \text{Log}[a + b x]}{b^3} - \frac{2 \text{Cos}[a + b x] \text{Sin}[a + b x]}{3 b^3} - \frac{\text{Sin}[2 a + 2 b x]}{12 b^3} - \frac{2 \left(\left(2 - a^2 + a b x - b^2 x^2\right) \text{Cos}[a + b x] - (a - 2 b x) \text{Sin}[a + b x]\right) \text{SinIntegral}[a + b x]}{3 b^3} + \frac{1}{3} \left(\frac{a^3}{b^3} + x^3\right) \text{SinIntegral}[a + b x]^2 + \frac{\left(2 - 3 a^2\right) \text{SinIntegral}[2 (a + b x)]}{3 b^3}$$

- *Mathematica* is unable integrate x^m SinIntegral[a + b x]^2 for integer $m > 0$:

$$\int \text{SinIntegral}[a + b x]^2 \, dx$$

$$\frac{2 \text{Cos}[a + b x] \text{SinIntegral}[a + b x] + (a + b x) \text{SinIntegral}[a + b x]^2 - \text{SinIntegral}[2 (a + b x)]}{b}$$

$$\int x \, \text{SinIntegral}[a + b x]^2 \, dx$$

$$\int x \, \text{SinIntegral}[a + b x]^2 \, dx$$

$$\int x^2 \, \text{SinIntegral}[a + b x]^2 \, dx$$

$$\int x^2\,\text{SinIntegral}[a+b\,x]^2\,dx$$

- *Maple* is unable integrate x^m $\text{SinIntegral}[a+b\,x]^2$ for integer $m>1$:

```
int (Si (a + b * x) ^2, x);
```

$$\frac{2\,\text{Cos}[a+b\,x]\,\text{Si}[a+b\,x]+(a+b\,x)\,\text{Si}[a+b\,x]^2-\text{Si}[2\,(a+b\,x)]}{b}$$

```
int (x * Si (a + b * x) ^2, x);
```

$$-\frac{\text{Ci}[2\,(a+b\,x)]}{2\,b^2}+\frac{\text{Cos}[2\,a+2\,b\,x]}{4\,b^2}+\frac{\text{Log}[a+b\,x]}{2\,b^2}-\frac{1}{2}\left(\frac{a^2}{b^2}-x^2\right)\text{Si}[a+b\,x]^2+$$
$$\frac{a\,\text{Si}[2\,(a+b\,x)]}{b^2}-\frac{\text{Si}[a+b\,x]\,((a-b\,x)\,\text{Cos}[a+b\,x]+\text{Sin}[a+b\,x])}{b^2}$$

```
int (x^2 * Si (a + b * x) ^2, x);
```

$$\int x^2\,\text{Ei}[a+b\,x]^2\,dx$$

Note that these systems give similar results to the above for the cosine integral, hyperbolic sine integral and hyperbolic cosine integral functions.

Integral Function Integration Problem 3

$$\int x^m \mathrm{Cos}[b\,x]\,\mathrm{SinIntegral}[b\,x]\,dx$$

- *Rubi* is able to integrate x^m **Cos[b x] SinIntegral[b x]** for all integer m except -1:

Int[x^2 Cos[b x] SinIntegral[b x], x]

$$-\frac{x^2}{4\,b}-\frac{\mathrm{CosIntegral}[2\,b\,x]}{b^3}+\frac{\mathrm{Log}[x]}{b^3}+\frac{x\,\mathrm{Cos}[b\,x]\,\mathrm{Sin}[b\,x]}{2\,b^2}-\frac{5\,\mathrm{Sin}[b\,x]^2}{4\,b^3}+\frac{2\,x\,\mathrm{Cos}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{b^2}-\frac{2\,\mathrm{Sin}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{b^3}+\frac{x^2\,\mathrm{Sin}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{b}$$

Int[x Cos[b x] SinIntegral[b x], x]

$$-\frac{x}{2\,b}+\frac{\mathrm{Cos}[b\,x]\,\mathrm{Sin}[b\,x]}{2\,b^2}+\frac{\mathrm{Cos}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{b^2}+\frac{x\,\mathrm{Sin}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{b}-\frac{\mathrm{SinIntegral}[2\,b\,x]}{2\,b^2}$$

Int[Cos[b x] SinIntegral[b x], x]

$$\frac{\mathrm{CosIntegral}[2\,b\,x]}{2\,b}-\frac{\mathrm{Log}[x]}{2\,b}+\frac{\mathrm{Sin}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{b}$$

$$\mathbf{Int}\left[\frac{\mathbf{Cos}[b\,x]\,\mathbf{SinIntegral}[b\,x]}{x},\,x\right]$$

$$\mathrm{Int}\left[\frac{\mathrm{Cos}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{x},\,x\right]$$

$$\mathbf{Int}\left[\frac{\mathbf{Cos}[b\,x]\,\mathbf{SinIntegral}[b\,x]}{x^2},\,x\right]$$

$$b\,\mathrm{CosIntegral}[2\,b\,x]-\frac{\mathrm{Sin}[2\,b\,x]}{2\,x}-\frac{\mathrm{Cos}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{x}-\frac{1}{2}\,b\,\mathrm{SinIntegral}[b\,x]^2$$

$$\mathbf{Int}\left[\frac{\mathbf{Cos}[b\,x]\,\mathbf{SinIntegral}[b\,x]}{x^3},\,x\right]$$

$$\frac{b}{4\,x}-\frac{b\,\mathrm{Cos}[2\,b\,x]}{2\,x}-\frac{1}{2}\,b^2\,\mathrm{Int}\left[\frac{\mathrm{Cos}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{x},\,x\right]-\frac{\mathrm{Sin}[2\,b\,x]}{8\,x^2}-\frac{\mathrm{Cos}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{2\,x^2}+\frac{b\,\mathrm{Sin}[b\,x]\,\mathrm{SinIntegral}[b\,x]}{2\,x}-b^2\,\mathrm{SinIntegral}[2\,b\,x]$$

- *Mathematica* is not able to integrate x^m **Cos[b x] SinIntegral[b x]** for negative integer m:

$$\int x^2\,\mathbf{Cos}[b\,x]\,\mathbf{SinIntegral}[b\,x]\,dx$$

$$\frac{1}{8\,b^3}\left(-2\,b^2\,x^2+5\,\text{Cos}[2\,b\,x]-8\,\text{CosIntegral}[2\,b\,x]+8\,\text{Log}[x]+2\,b\,x\,\text{Sin}[2\,b\,x]+8\left(2\,b\,x\,\text{Cos}[b\,x]+\left(-2+b^2\,x^2\right)\text{Sin}[b\,x]\right)\text{SinIntegral}[b\,x]\right)$$

$$\int x\,\text{Cos}[b\,x]\,\text{SinIntegral}[b\,x]\,dx$$

$$\frac{-2\,b\,x+\text{Sin}[2\,b\,x]+4\,(\text{Cos}[b\,x]+b\,x\,\text{Sin}[b\,x])\,\text{SinIntegral}[b\,x]-2\,\text{SinIntegral}[2\,b\,x]}{4\,b^2}$$

$$\int \text{Cos}[b\,x]\,\text{SinIntegral}[b\,x]\,dx$$

$$\frac{\text{CosIntegral}[2\,b\,x]}{2\,b}-\frac{\text{Log}[b\,x]}{2\,b}+\frac{\text{Sin}[b\,x]\,\text{SinIntegral}[b\,x]}{b}$$

$$\int \frac{\text{Cos}[b\,x]\,\text{SinIntegral}[b\,x]}{x}\,dx$$

$$\int \frac{\text{Cos}[b\,x]\,\text{SinIntegral}[b\,x]}{x}\,dx$$

$$\int \frac{\text{Cos}[b\,x]\,\text{SinIntegral}[b\,x]}{x^2}\,dx$$

$$\int \frac{\text{Cos}[b\,x]\,\text{SinIntegral}[b\,x]}{x^2}\,dx$$

$$\int \frac{\text{Cos}[b\,x]\,\text{SinIntegral}[b\,x]}{x^3}\,dx$$

$$\int \frac{\text{Cos}[b\,x]\,\text{SinIntegral}[b\,x]}{x^3}\,dx$$

- *Maple* is not able to integrate $\mathbf{x^m\,Cos\,(b\,x)\,Si\,(b\,x)}$ for negative integer m:

```
int (x^2 * cos (b * x) * Si (b * x), x);
```

$$-\frac{x^2}{4\,b}-\frac{\text{Ci}[2\,b\,x]}{b^3}+\frac{\text{Log}[x]}{b^3}+\frac{x\,\text{Cos}[b\,x]\,\text{Sin}[b\,x]}{2\,b^2}-\frac{5\,\text{Sin}[b\,x]^2}{4\,b^3}+\frac{2\,x\,\text{Cos}[b\,x]\,\text{Si}[b\,x]}{b^2}-\frac{2\,\text{Sin}[b\,x]\,\text{Si}[b\,x]}{b^3}+\frac{x^2\,\text{Sin}[b\,x]\,\text{Si}[b\,x]}{b}$$

```
int (x * cos (b * x) * Si (b * x), x);
```

$$-\frac{x}{2\,b}+\frac{\text{Cos}[b\,x]\,\text{Sin}[b\,x]}{2\,b^2}+\frac{\text{Cos}[b\,x]\,\text{Si}[b\,x]}{b^2}+\frac{x\,\text{Sin}[b\,x]\,\text{Si}[b\,x]}{b}-\frac{\text{Si}[2\,b\,x]}{2\,b^2}$$

```
int (cos (b * x) * Si (b * x), x);
```

$$\frac{\text{Ci}[2\,b\,x]}{2\,b}-\frac{\text{Log}[x]}{2\,b}+\frac{\text{Sin}[b\,x]\,\text{Si}[b\,x]}{b}$$

```
int (cos (b * x) * Si (b * x) / x, x);
```

$$\int \frac{\text{Cos}[b\,x]\,\text{Si}[b\,x]}{x}\,dx$$

```
int (cos (b * x) * Si (b * x) / x^2, x);
```

$$\int \frac{\text{Cos}[b\,x]\,\text{Si}[b\,x]}{x^2}\,dx$$

```
int (cos (b * x) * Si (b * x) / x^3, x);
```

$$\int \frac{\text{Cos}[b\,x]\,\text{Si}[b\,x]}{x^3}\,dx$$

Note that these systems give similar results to the above for the cosine integral, hyperbolic sine integral and hyperbolic cosine integral functions.

Integral Function Integration Problem 4

$$\int \text{CosIntegral}[a+b\,x]\,\text{Sin}[c+d\,x]\,dx$$

- The *Rubi* result is in terms of trig integral functions:

```
Int[CosIntegral[a + b x] Sin[c + d x], x]
```

$$-\frac{\text{Cos}[c+d\,x]\,\text{CosIntegral}[a+b\,x]}{d}+\frac{\text{Cos}\left[c-\frac{a\,d}{b}\right]\text{CosIntegral}\left[\frac{(b-d)\,(a+b\,x)}{b}\right]}{2\,d}+\frac{\text{Cos}\left[c-\frac{a\,d}{b}\right]\text{CosIntegral}\left[\frac{(b+d)\,(a+b\,x)}{b}\right]}{2\,d}+\frac{\text{Sin}\left[c-\frac{a\,d}{b}\right]\text{SinIntegral}\left[\frac{(b-d)\,(a+b\,x)}{b}\right]}{2\,d}-\frac{\text{Sin}\left[c-\frac{a\,d}{b}\right]\text{SinIntegral}\left[\frac{(b+d)\,(a+b\,x)}{b}\right]}{2\,d}$$

- The *Mathematica* result is in terms of exponential integral functions:

$$\int \text{CosIntegral}[a+b\,x]\,\text{Sin}[c+d\,x]\,dx$$

$$-\frac{\text{Cos}[c+d\,x]\,\text{CosIntegral}[a+b\,x]}{d}+\frac{e^{i\left(c-\frac{a\,d}{b}\right)}\text{ExpIntegralEi}\left[-\frac{i\,(b-d)\,(a+b\,x)}{b}\right]}{4\,d}+\frac{e^{-i\left(c-\frac{a\,d}{b}\right)}\text{ExpIntegralEi}\left[\frac{i\,(b-d)\,(a+b\,x)}{b}\right]}{4\,d}+\frac{e^{-i\left(c-\frac{a\,d}{b}\right)}\text{ExpIntegralEi}\left[-\frac{i\,(b+d)\,(a+b\,x)}{b}\right]}{4\,d}+\frac{e^{i\left(c-\frac{a\,d}{b}\right)}\text{ExpIntegralEi}\left[\frac{i\,(b+d)\,(a+b\,x)}{b}\right]}{4\,d}$$

- The *Maple* result is in terms of trig integral functions:

```
int(Ci(a + b * x) * sin(c + d * x), x);
```

$$-\frac{\text{Cos}[c+d\,x]\,\text{Ci}[a+b\,x]}{d}+\frac{\text{Cos}\left[c-\frac{a\,d}{b}\right]\text{Ci}\left[\frac{(b-d)\,(a+b\,x)}{b}\right]}{2\,d}+\frac{\text{Cos}\left[c-\frac{a\,d}{b}\right]\text{Ci}\left[\frac{(b+d)\,(a+b\,x)}{b}\right]}{2\,d}+\frac{\text{Sin}\left[c-\frac{a\,d}{b}\right]\text{Si}\left[\frac{(b-d)\,(a+b\,x)}{b}\right]}{2\,d}-\frac{\text{Sin}\left[c-\frac{a\,d}{b}\right]\text{Si}\left[\frac{(b+d)\,(a+b\,x)}{b}\right]}{2\,d}$$

Note that these systems give similar results to the above for the sine integral function.