Trig Function Integration Problem 1

$$\int \frac{1}{1+\text{Sin}[a+b\,x]}\,dx$$

- The *Rubi* results are simple and symmetric:

$$\text{Int}\left[\frac{1}{1+\text{Sin}[a+b\,x]},\,x\right]$$

$$-\frac{\text{Cos}[a+b\,x]}{b\,(1+\text{Sin}[a+b\,x])}$$

$$\text{Int}\left[\frac{1}{1-\text{Sin}[a+b\,x]},\,x\right]$$

$$-\frac{\text{Cos}[a+b\,x]}{b\,(-1+\text{Sin}[a+b\,x])}$$

- The *Mathematica* results are more complicated and not symmetric:

$$\int \frac{1}{1+\text{Sin}[a+b\,x]}\,dx$$

$$\frac{2\,\text{Sin}\left[\frac{1}{2}\,(a+b\,x)\right]\,\left(\text{Cos}\left[\frac{1}{2}\,(a+b\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{b\,(1+\text{Sin}[a+b\,x])}$$

$$\int \frac{1}{1-\text{Sin}[a+b\,x]}\,dx$$

$$\frac{2\,\text{Sin}\left[\frac{1}{2}\,(a+b\,x)\right]}{b\,\left(\text{Cos}\left[\frac{1}{2}\,(a+b\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}$$

- The *Maple* results are simple and symmetric:

```
int(1/(1+sin(a+b*x)),x);
```

$$-\frac{2}{b\,\left(1+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}$$

```
int(1/(1-sin(a+b*x)),x);
```

$$-\frac{2}{b\,\left(-1+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}$$

Trig Function Integration Problem 2

$$\int \frac{1}{1 + \operatorname{Sec}[a + b\,x]}\,dx$$

- The *Rubi* result is simple:

```
Int[1/(1 + Sec[a + b x]), x]
```

$$x - \frac{\operatorname{Sin}[a + b\,x]}{b\,(1 + \operatorname{Cos}[a + b\,x])}$$

- The *Mathematica* result is unnecessarily complicated:

$$\int \frac{1}{1 + \operatorname{Sec}[a + b\,x]}\,dx$$

$$\frac{2\operatorname{Cos}\left[\frac{1}{2}(a + b\,x)\right]\operatorname{Sec}[a + b\,x]\left((a + b\,x)\operatorname{Cos}\left[\frac{1}{2}(a + b\,x)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + b\,x)\right]\right)}{b\,(1 + \operatorname{Sec}[a + b\,x])}$$

- The *Maple* result includes a term whose derivative is 1, the same as the derivative of x:

```
int(1/(1+sec(a+b*x)), x);
```

$$-\frac{\operatorname{Tan}\left[\frac{1}{2}(a + b\,x)\right]}{b} + \frac{2\operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{1}{2}(a + b\,x)\right]\right]}{b}$$

Trig Function Integration Problem 3

$$\int \text{Sin}[x]\ \text{Tan}[n\ x]\ dx$$

- The *Rubi* results are simple, expressed in trigonometric form and grow modestly with n:

```
Int[Sin[x] Tan[x], x]
```

$\text{ArcTanh}[\text{Sin}[x]] - \text{Sin}[x]$

```
Int[Sin[x] Tan[2 x], x]
```

$$\frac{\text{ArcTanh}\left[\sqrt{2}\ \text{Sin}[x]\right]}{\sqrt{2}} - \text{Sin}[x]$$

```
Int[Sin[x] Tan[3 x], x]
```

$$\frac{1}{3}\ \text{ArcTanh}\left[\frac{3\ \text{Sin}[x]}{1+2\ \text{Sin}[x]^2}\right] - \text{Sin}[x]$$

```
Int[Sin[x] Tan[4 x], x]
```

$$\frac{1}{4}\sqrt{2-\sqrt{2}}\ \text{ArcTanh}\left[\frac{2\ \text{Sin}[x]}{\sqrt{2-\sqrt{2}}}\right] + \frac{1}{4}\sqrt{2+\sqrt{2}}\ \text{ArcTanh}\left[\frac{2\ \text{Sin}[x]}{\sqrt{2+\sqrt{2}}}\right] - \text{Sin}[x]$$

- The *Mathematica* results grow unpredictably, are expressed in terms of logarithms and not in closed-form when n is 4:

$$\int \text{Sin}[x]\ \text{Tan}[x]\ dx$$

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Sin}[x]$$

$$\int \text{Sin}[x]\ \text{Tan}[2\ x]\ dx$$

$$\frac{1}{8}\left(-2\ i\ \sqrt{2}\ \text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right] - \left(-1+\sqrt{2}\right)\text{Sin}\left[\frac{x}{2}\right]}{\left(1+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]}\right] - \right.$$
$$2\ i\ \sqrt{2}\ \text{ArcTan}\left[\frac{\text{Cos}\left[\frac{x}{2}\right] - \left(1+\sqrt{2}\right)\text{Sin}\left[\frac{x}{2}\right]}{\left(-1+\sqrt{2}\right)\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]}\right] + 2\sqrt{2}\ \text{Log}\left[\sqrt{2} + 2\ \text{Sin}[x]\right] -$$
$$\left.\sqrt{2}\ \text{Log}\left[4+2\sqrt{2}\ \text{Cos}[x] - 2\sqrt{2}\ \text{Sin}[x]\right] - \sqrt{2}\ \text{Log}\left[-2\left(-2+\sqrt{2}\ \text{Cos}[x] + \sqrt{2}\ \text{Sin}[x]\right)\right] - 8\ \text{Sin}[x]\right)$$

$$\int \text{Sin}[x]\ \text{Tan}[3\ x]\ dx$$

$$-\frac{1}{3}\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{1}{3}\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]-\frac{1}{6}\text{Log}[-1+2\,\text{Sin}[x]]+\frac{1}{6}\text{Log}[1+2\,\text{Sin}[x]]-\text{Sin}[x]$$

$$\int \textbf{Sin}[\textbf{x}]\ \textbf{Tan}[\textbf{4 x}]\ \mathbb{d}\textbf{x}$$

$$\frac{1}{16}\text{RootSum}\Big[1+\#1^8\ \&,\ \frac{2\,\text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x]-\#1}\right]-\mathbb{i}\,\text{Log}\left[1-2\,\text{Cos}[x]\,\#1+\#1^2\right]+2\,\text{ArcTan}\left[\frac{\text{Sin}[x]}{\text{Cos}[x]-\#1}\right]\#1^6-\mathbb{i}\,\text{Log}\left[1-2\,\text{Cos}[x]\,\#1+\#1^2\right]\#1^6}{\#1^7}\ \&\Big]-\text{Sin}[x]$$

- The *Maple* results are simple, but expressed in terms of logarithms when n is odd:

```
int (sin (x) * tan (x), x);
```

$$\text{Log}[\text{Sec}[x]+\text{Tan}[x]]-\text{Sin}[x]$$

```
int (sin (x) * tan (2 * x), x);
```

$$\frac{\text{ArcTanh}\left[\sqrt{2}\ \text{Sin}[x]\right]}{\sqrt{2}}-\text{Sin}[x]$$

```
int (sin (x) * tan (3 * x), x);
```

$$-\frac{1}{6}\text{Log}[-1+\text{Sin}[x]]+\frac{1}{6}\text{Log}[1+\text{Sin}[x]]-\frac{1}{6}\text{Log}[-1+2\,\text{Sin}[x]]+\frac{1}{6}\text{Log}[1+2\,\text{Sin}[x]]-\text{Sin}[x]$$

```
int (sin (x) * tan (4 * x), x);
```

$$\frac{1}{4}\sqrt{2-\sqrt{2}}\ \text{ArcTanh}\left[\frac{2\,\text{Sin}[x]}{\sqrt{2-\sqrt{2}}}\right]+\frac{1}{4}\sqrt{2+\sqrt{2}}\ \text{ArcTanh}\left[\frac{2\,\text{Sin}[x]}{\sqrt{2+\sqrt{2}}}\right]-\text{Sin}[x]$$

Note that these systems give similar results to the above for the cosine function.

Trig Function Integration Problem 4

$$\int \sqrt{a + b\,\text{Sin}[x]}\,dx$$

- *Rubi* instantaneously computes the general case, and the special cases are relatively simple:

```
Int[Sqrt[a + b Sin[x]], x]
```

$$\frac{2\,\text{EllipticE}\left[\frac{1}{4}(-\pi + 2x), \frac{2b}{a+b}\right]\sqrt{a + b\,\text{Sin}[x]}}{\sqrt{\frac{a+b\,\text{Sin}[x]}{a+b}}}$$

```
Int[Sqrt[1 + Sin[x]], x]
```

$$-\frac{2\,\text{Cos}[x]}{\sqrt{1 + \text{Sin}[x]}}$$

```
Int[Sqrt[1 - Sin[x]], x]
```

$$\frac{2\,\text{Cos}[x]}{\sqrt{1 - \text{Sin}[x]}}$$

- *Mathematica* requires 24 seconds to compute the general case, and the special cases are more complicated:

$$\int \sqrt{a + b\,\text{Sin}[x]}\,dx$$

$$-\frac{2\,\text{EllipticE}\left[\frac{1}{4}(\pi - 2x), \frac{2b}{a+b}\right]\sqrt{a + b\,\text{Sin}[x]}}{\sqrt{\frac{a+b\,\text{Sin}[x]}{a+b}}}$$

$$\int \sqrt{1 + \text{Sin}[x]}\,dx$$

$$\frac{2\left(-\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)\sqrt{1 + \text{Sin}[x]}}{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]}$$

$$\int \sqrt{1 - \text{Sin}[x]}\,dx$$

$$\frac{2\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)\sqrt{1 - \text{Sin}[x]}}{\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]}$$

- The *Maple* result for the general case is more complicated (note that *Maple* defines EllipticE and F differently than *Mathmatica*):

```
int (sqrt (a + b * sin (x)), x);
```

$$\frac{1}{b\,\mathrm{Cos}[x]\,\sqrt{a+b\,\mathrm{Sin}[x]}}2\,(a-b)\,\sqrt{\frac{a+b\,\mathrm{Sin}[x]}{a-b}}\,\sqrt{\frac{b\,(1-\mathrm{Sin}[x])}{a+b}}\,\sqrt{-\frac{b\,(1+\mathrm{Sin}[x])}{a-b}}\left((a+b)\,\mathrm{EllipticF}\left[\sqrt{\frac{a+b\,\mathrm{Sin}[x]}{a-b}},\sqrt{\frac{a-b}{a+b}}\right]-(a+b)\,\mathrm{EllipticE}\left[\sqrt{\frac{a+b\,\mathrm{Sin}[x]}{a-b}},\sqrt{\frac{a-b}{a+b}}\right]\right)$$

```
int (sqrt (1 + sin (x)), x);
```

$$\frac{2\,(\mathrm{Sin}[x]-1)\,\sqrt{1+\mathrm{Sin}[x]}}{\mathrm{Cos}[x]}$$

```
int (sqrt (1 - sin (x)), x);
```

$$\frac{2\,\mathrm{Cos}[x]}{\sqrt{1-\mathrm{Sin}[x]}}$$

Trig Function Integration Problem 5

$$\int \mathrm{Sec}[a + b\,x]\, dx \quad \& \quad \int \sqrt{\mathrm{Sec}[a + b\,x]^2}\, dx$$

- The *Rubi* results are symmetric and simple:

```
Int[Sec[a + b x], x]
```

$$\frac{\mathrm{ArcTanh}[\mathrm{Sin}[a + b\,x]]}{b}$$

```
Int[Sqrt[Sec[a + b x]^2], x]
```

$$\frac{\mathrm{ArcSinh}[\mathrm{Tan}[a + b\,x]]}{b}$$

- The *Mathematica* results are symmetric but *not* simple:

$$\int \mathrm{Sec}[a + b\,x]\, dx$$

$$-\frac{\mathrm{Log}\left[\mathrm{Cos}\left[\frac{a}{2} + \frac{b\,x}{2}\right] - \mathrm{Sin}\left[\frac{a}{2} + \frac{b\,x}{2}\right]\right]}{b} + \frac{\mathrm{Log}\left[\mathrm{Cos}\left[\frac{a}{2} + \frac{b\,x}{2}\right] + \mathrm{Sin}\left[\frac{a}{2} + \frac{b\,x}{2}\right]\right]}{b}$$

$$\int \sqrt{\mathrm{Sec}[a + b\,x]^2}\, dx$$

$$-\frac{1}{b} \mathrm{Cos}[a + b\,x] \left(\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(a + b\,x)\right] - \mathrm{Sin}\left[\frac{1}{2}(a + b\,x)\right]\right] - \mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(a + b\,x)\right] + \mathrm{Sin}\left[\frac{1}{2}(a + b\,x)\right]\right]\right) \sqrt{\mathrm{Sec}[a + b\,x]^2}$$

- The *Maple* results are relatively simple but *not* symmetric:

```
int(sec(a + b * x), x);
```

$$\frac{\mathrm{Log}[\mathrm{Sec}[a + b\,x] + \mathrm{Tan}[a + b\,x]]}{b}$$

```
int(sqrt(sec(a + b * x)^2), x);
```

$$-\frac{2\,\mathrm{ArcTanh}[(-1 + \mathrm{Cos}[a + b\,x])\,\mathrm{Csc}[a + b\,x]]\,\mathrm{Cos}[a + b\,x]\,\sqrt{\mathrm{Sec}[a + b\,x]^2}}{b}$$

Trig Function Integration Problem 6

$$\int \sqrt{\mathrm{Sec}[x]^2}\, dx \quad \& \quad \int \sqrt{\mathrm{Csc}[x]^2}\, dx$$

- The *Rubi* results are simple and symmetric:

```
Int[Sqrt[Sec[x]^2], x]
```

ArcSinh[Tan[x]]

```
Int[Sqrt[Csc[x]^2], x]
```

-ArcCsch[Tan[x]]

- The *Mathematica* results are symmetric but more complicated:

$$\int \sqrt{\mathrm{Sec}[x]^2}\, dx$$

$$\mathrm{Cos}[x] \left(-\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] - \mathrm{Sin}\left[\frac{x}{2}\right]\right] + \mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right] + \mathrm{Sin}\left[\frac{x}{2}\right]\right]\right) \sqrt{\mathrm{Sec}[x]^2}$$

$$\int \sqrt{\mathrm{Csc}[x]^2}\, dx$$

$$\sqrt{\mathrm{Csc}[x]^2} \left(-\mathrm{Log}\left[2\,\mathrm{Cos}\left[\frac{x}{2}\right]\right] + \mathrm{Log}\left[2\,\mathrm{Sin}\left[\frac{x}{2}\right]\right]\right) \mathrm{Sin}[x]$$

- The *Maple* results are *not* symmetric and more complicated:

```
int(sqrt(sec(x)^2), x);
```

$$-2\,\mathrm{ArcTanh}[(-1 + \mathrm{Cos}[x])\,\mathrm{Csc}[x]]\,\mathrm{Cos}[x] \sqrt{\mathrm{Sec}[x]^2}$$

```
int(sqrt(csc(x)^2), x);
```

$$\frac{1}{2} \sqrt{4} \sqrt{\frac{1}{1 - \mathrm{Cos}[x]^2}}\, \mathrm{Sin}[x]\, \mathrm{Log}\left[\frac{1 - \mathrm{Cos}[x]}{\mathrm{Sin}[x]}\right]$$

Trig Function Integration Problem 7

$$\int\sqrt{1+\mathrm{Sec}[x]}\,dx \quad \& \quad \int\sqrt{1+\mathrm{Csc}[x]}\,dx$$

- The *Rubi* results are simple, expressed in terms of elementary functions and symmetric:

```
Int[Sqrt[1 + Sec[x]], x]
```

$$\frac{2\,\mathrm{ArcTanh}\left[\sqrt{1-\mathrm{Sec}[x]}\right]\mathrm{Tan}[x]}{\sqrt{1-\mathrm{Sec}[x]}\,\sqrt{1+\mathrm{Sec}[x]}}$$

```
Int[Sqrt[1 + Csc[x]], x]
```

$$-\frac{2\,\mathrm{ArcTanh}\left[\sqrt{1-\mathrm{Csc}[x]}\right]\mathrm{Cot}[x]}{\sqrt{1-\mathrm{Csc}[x]}\,\sqrt{1+\mathrm{Csc}[x]}}$$

- The *Mathematica* results are large, not expressed in terms of elementary functions and *not* symmetric:

$$\int\sqrt{1+\mathrm{Sec}[x]}\,dx$$

$$\left(4\sqrt{2}\left(3+2\sqrt{2}\right)\sqrt{\frac{-1+\sqrt{2}-\left(-2+\sqrt{2}\right)\mathrm{Cos}\left[\frac{x}{2}\right]}{1+\mathrm{Cos}\left[\frac{x}{2}\right]}}\left(1-\sqrt{2}+\left(-2+\sqrt{2}\right)\mathrm{Cos}\left[\frac{x}{2}\right]\right)\right.$$
$$\left(-1+\sqrt{2}+\left(-2+\sqrt{2}\right)\mathrm{Cos}\left[\frac{x}{2}\right]\right)^2\left(\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\mathrm{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],17-12\sqrt{2}\right]+\right.$$
$$\left.2\,\mathrm{EllipticPi}\left[-3+2\sqrt{2},-\mathrm{ArcSin}\left[\frac{\mathrm{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],17-12\sqrt{2}\right]\right)$$
$$\left.\mathrm{Sec}\left[\frac{x}{2}\right]\mathrm{Sec}[x]\sqrt{1+\mathrm{Sec}[x]}\right)\Big/\left(\sqrt{-\left(-1+\sqrt{2}+\left(-2+\sqrt{2}\right)\mathrm{Cos}\left[\frac{x}{2}\right]\right)\mathrm{Sec}\left[\frac{x}{4}\right]^2}\right)$$

$$\int\sqrt{1+\mathrm{Csc}[x]}\,dx$$

$$-\frac{2\,\mathrm{ArcTan}\left[\sqrt{-1+\mathrm{Csc}[x]}\right]\mathrm{Cot}[x]}{\sqrt{-1+\mathrm{Csc}[x]}\,\sqrt{1+\mathrm{Csc}[x]}}$$

- The *Maple* results are expressed in terms of elementary functions, but large and *not* symmetric:

```
int(sqrt(1 + sec(x)), x);
```

$$-2\,\mathrm{ArcTanh}\left[\sqrt{-\frac{\mathrm{Cos}[x]}{1+\mathrm{Cos}[x]}}\,\mathrm{Tan}[x]\right]\sqrt{-\frac{\mathrm{Cos}[x]}{1+\mathrm{Cos}[x]}}\,\sqrt{\frac{1+\mathrm{Cos}[x]}{\mathrm{Cos}[x]}}$$

```
int (sqrt (1 + csc (x)), x);
```

$$-\frac{1}{2\,(-1+\mathrm{Cos}[x]-\mathrm{Sin}[x])}\sqrt{2}\sqrt{\frac{1+\mathrm{Sin}[x]}{\mathrm{Sin}[x]}}\;\mathrm{Sin}[x]\sqrt{\frac{1-\mathrm{Cos}[x]}{\mathrm{Sin}[x]}}$$

$$\left(\mathrm{Log}\left[\frac{-1+\mathrm{Cos}[x]-\sqrt{\frac{1-\mathrm{Cos}[x]}{\mathrm{Sin}[x]}}\sqrt{2}\,\mathrm{Sin}[x]-\mathrm{Sin}[x]}{-1+\mathrm{Cos}[x]+\sqrt{\frac{1-\mathrm{Cos}[x]}{\mathrm{Sin}[x]}}\sqrt{2}\,\mathrm{Sin}[x]-\mathrm{Sin}[x]}\right]+4\,\mathrm{ArcTan}\left[\sqrt{\frac{1-\mathrm{Cos}[x]}{\mathrm{Sin}[x]}}\sqrt{2}+1\right]+\right.$$

$$\left.4\,\mathrm{ArcTan}\left[\sqrt{\frac{1-\mathrm{Cos}[x]}{\mathrm{Sin}[x]}}\sqrt{2}-1\right]+\mathrm{Log}\left[\frac{-1+\mathrm{Cos}[x]+\sqrt{\frac{1-\mathrm{Cos}[x]}{\mathrm{Sin}[x]}}\sqrt{2}\,\mathrm{Sin}[x]-\mathrm{Sin}[x]}{-1+\mathrm{Cos}[x]-\sqrt{\frac{1-\mathrm{Cos}[x]}{\mathrm{Sin}[x]}}\sqrt{2}\,\mathrm{Sin}[x]-\mathrm{Sin}[x]}\right]\right)$$

Trig Function Integration Problem 8

$$\int \sqrt{a \operatorname{Cos}[x] + b \operatorname{Sin}[x]} \, dx$$

- *Rubi* returns a relatively simple result by using the identity a cos(z)+b sin(z) = sqrt(a^2+b^2) cos(z-arctan(a,b)):

```
Int[Sqrt[c * Cos[x - d]], x]
```

$$-\frac{2 \sqrt{c \operatorname{Cos}[d-x]} \operatorname{EllipticE}\left[\frac{d-x}{2}, 2\right]}{\sqrt{\operatorname{Cos}[d-x]}}$$

```
Int[Sqrt[a Cos[x] + b Sin[x]], x]
```

$$\frac{2 \operatorname{EllipticE}\left[\frac{1}{2} (x - \operatorname{ArcTan}[a, b]), 2\right] \sqrt{a \operatorname{Cos}[x] + b \operatorname{Sin}[x]}}{\sqrt{\frac{a \operatorname{Cos}[x]+b \operatorname{Sin}[x]}{\sqrt{a^2+b^2}}}}$$

- *Mathematica* knows the identity, but instead returns a complicated expression involving a hypergeometric function:

$$\int \sqrt{c * \operatorname{Cos}[x - d]} \, dx$$

$$-\frac{2 \sqrt{c \operatorname{Cos}[d-x]} \operatorname{EllipticE}\left[\frac{d-x}{2}, 2\right]}{\sqrt{\operatorname{Cos}[d-x]}}$$

$$\int \sqrt{a \operatorname{Cos}[x] + b \operatorname{Sin}[x]} \, dx$$

$$\left(-b \left(a^2+b^2\right) \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2\right] \operatorname{Sin}\left[x - \operatorname{ArcTan}\left[\frac{b}{a}\right]\right] + \sqrt{\operatorname{Sin}\left[x - \operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2} \left(2 a^3 \sqrt{1+\frac{b^2}{a^2}} \operatorname{Cos}[x] - 2 a \left(a^2+b^2\right) \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{b}{a}\right]\right] + 2 a^2 b \sqrt{1+\frac{b^2}{a^2}} \operatorname{Sin}[x] + a^2 b \operatorname{Sin}\left[x - \operatorname{ArcTan}\left[\frac{b}{a}\right]\right] + b^3 \operatorname{Sin}\left[x - \operatorname{ArcTan}\left[\frac{b}{a}\right]\right]\right)\right) \Big/ \left(a b \sqrt{1+\frac{b^2}{a^2}} \sqrt{a \operatorname{Cos}[x] + b \operatorname{Sin}[x]} \sqrt{\operatorname{Sin}\left[x - \operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2}\right)$$

- The *Maple* result is huge (note that *Maple* defines EllipticE differently than *Mathmatica*):

```
leafcount(int(sqrt(c * cos(x - d)), x));
```

$$-\frac{2\,c\,\sqrt{1-\text{Cos}\left[\frac{d-x}{2}\right]^2}\,\sqrt{1-2\,\text{Cos}\left[\frac{d-x}{2}\right]^2}\,\text{EllipticE}\left[\text{Cos}\left[\frac{d-x}{2}\right],\sqrt{2}\right]}{c\,\sqrt{-1+2\,\text{Cos}\left[\frac{d-x}{2}\right]^2}\,\text{Sin}\left[\frac{d-x}{2}\right]}$$

```
leafcount (int (sqrt (a * cos (x) + b * sin (x)), x));
```

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Trig Function Integration Problem 9

$$\int \frac{\text{Sin}[x]}{\sqrt{a + b\,\text{Sin}[x]^2}}\,dx$$

- The *Rubi* results are simple and expressed in trigonometric form:

$$\text{Int}\left[\frac{\text{Sin}[x]}{\sqrt{a + b\,\text{Sin}[x]^2}}, x\right]$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a+b-b\,\text{Cos}[x]^2}\,\text{Sec}[x]}{\sqrt{b}}\right]}{\sqrt{b}}$$

$$\text{Int}\left[\frac{\text{Sin}[x]}{\sqrt{1 + \text{Sin}[x]^2}}, x\right]$$

$$-\text{ArcSin}\left[\frac{\text{Cos}[x]}{\sqrt{2}}\right]$$

$$\text{Int}\left[\frac{\text{Sin}[x]}{\sqrt{1 - \text{Sin}[x]^2}}, x\right]$$

$$-\frac{\text{Cos}[x]\,\text{Log}[\text{Cos}[x]]}{\sqrt{\text{Cos}[x]^2}}$$

- The *Mathematica* results are simple but can involve the imaginary unit:

$$\int \frac{\text{Sin}[x]}{\sqrt{a + b\,\text{Sin}[x]^2}}\,dx$$

$$-\frac{\text{Log}\left[\sqrt{2}\,\sqrt{-b}\,\text{Cos}[x] + \sqrt{2\,a + b - b\,\text{Cos}[2\,x]}\right]}{\sqrt{-b}}$$

$$\int \frac{\text{Sin}[x]}{\sqrt{1 + \text{Sin}[x]^2}}\,dx$$

$$i\,\text{Log}\left[i\,\sqrt{2}\,\text{Cos}[x] + \sqrt{3 - \text{Cos}[2\,x]}\right]$$

$$\int \frac{\text{Sin}[x]}{\sqrt{1 - \text{Sin}[x]^2}}\,dx$$

$$-\frac{\text{Cos}[x]\,\text{Log}[\text{Cos}[x]]}{\sqrt{\text{Cos}[x]^2}}$$

The *Maple* results are more complicated:

```
int (sin (x) / sqrt (a + b * sin (x) ^2), x);
```

$$\frac{\sqrt{-\left(a + b\,\mathrm{Sin}[x]^2\right)\left(-1 + \mathrm{Sin}[x]^2\right)}\ \mathrm{ArcTan}\left[\frac{2\,b\,\mathrm{Sin}[x]^2 + a - b}{2\sqrt{b}\ \sqrt{-\left(a + b\,\mathrm{Sin}[x]^2\right)\left(-1 + \mathrm{Sin}[x]^2\right)}}\right]}{2\sqrt{b}\ \mathrm{Cos}[x]\ \sqrt{a + b\,\mathrm{Sin}[x]^2}}$$

```
int (sin (x) / sqrt (1 + sin (x) ^2), x);
```

$$\frac{\sqrt{\mathrm{Cos}[x]^2\left(1 + \mathrm{Sin}[x]^2\right)}\ \mathrm{ArcSin}\left[\mathrm{Sin}[x]^2\right]}{2\,\mathrm{Cos}[x]\ \sqrt{1 + \mathrm{Sin}[x]^2}}$$

```
int (sin (x) / sqrt (1 - sin (x) ^2), x);
```

$$\frac{\left(-1 + \mathrm{Sin}[x]^2\right)\left(\mathrm{Log}[\mathrm{Sin}[x] - 1] + \mathrm{Log}[1 + \mathrm{Sin}[x]]\right)}{2\,\mathrm{Cos}[x]\ \sqrt{1 - \mathrm{Sin}[x]^2}}$$

Trig Function Integration Problem 10

$$\int \frac{\text{Cot}[x]}{\sqrt{a + b\,\text{Tan}[x]^2 + c\,\text{Tan}[x]^4}}\, dx$$

- *Rubi* is able to integrate the expression:

$$\text{Int}\left[\frac{\text{Cot}[x]}{\sqrt{a + b\,\text{Tan}[x]^2 + c\,\text{Tan}[x]^4}}, x\right]$$

$$-\frac{\text{ArcTanh}\left[\frac{2\sqrt{a}\,\sqrt{a+b\,\text{Tan}[x]^2+c\,\text{Tan}[x]^4}}{2\,a+b\,\text{Tan}[x]^2}\right]}{2\sqrt{a}} + \frac{\text{ArcTanh}\left[\frac{2\sqrt{a-b+c}\,\sqrt{a+b\,\text{Tan}[x]^2+c\,\text{Tan}[x]^4}}{2\,a-b+(b-2\,c)\,\text{Tan}[x]^2}\right]}{2\sqrt{a-b+c}}$$

- *Mathematica* is unable to integrate the expression in 60 seconds:

$$\int \frac{\text{Cot}[x]}{\sqrt{a + b\,\text{Tan}[x]^2 + c\,\text{Tan}[x]^4}}\, dx$$

$Aborted

- *Maple* is unable to integrate the expression:

```
int(cot(x)/sqrt(a+b*tan(x)^2+c*tan(x)^4), x);
```

$$\int \frac{\text{Cot}[x]}{\sqrt{a + b\,\text{Tan}[x]^2 + c\,\text{Tan}[x]^4}}\, dx$$