Integration Rules for $\int \left(\sin^j(z)\right)^m \left(A\left(\sin^k(z)\right)^p + B\left(\sin^k(z)\right)^{p+1} + C\left(\sin^k(z)\right)^{p+2}\right) dz$ when $j^2 = 1 \bigwedge k^2 = 1$

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1}\right) dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $j^2 = k^2 = 1 \bigwedge (j = k \bigvee p \in \mathbb{Z}) \bigwedge p \neq 1 \bigwedge p \neq -2$**, then**

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1}\right) dx \longrightarrow \int \left(\sin[c+d\,x]^j\right)^{m+j\,k\,p} \left(A + B\sin[c+d\,x]^k\right) dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_),x_Symbol] :=
  Int[(sin[c+d*x]^j)^(m+j*k*p)*(A+B*sin[c+d*x]^k),x] /;
FreeQ[{c,d,A,B,m,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[p+1-q] &&
  (ZeroQ[j-k] || IntegerQ[p]) && p≠-2
```

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1 \bigwedge m \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$**, then**

$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1}\right) dx \longrightarrow \int \left(\sin[c+d\,x]^k\right)^{p-m} \left(A + B\sin[c+d\,x]^k\right) dx$$

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```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_),x_Symbol] :=
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  Not[IntegerQ[p]]
```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z \left(\sqrt{\text{Sec}[z]}\ \sqrt{\text{Cos}[z]}\right) = 0$
- **Rule: If** $k^2 = 1 \bigwedge m - \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, **then**

$$\int \left(\sin[c + d\,x]^{-k}\right)^m \left(A \left(\sin[c + d\,x]^k\right)^p + B \left(\sin[c + d\,x]^k\right)^{p+1}\right) dx \longrightarrow$$
$$\sqrt{\text{Csc}[c + d\,x]}\ \sqrt{\text{Sin}[c + d\,x]} \int \left(\sin[c + d\,x]^k\right)^{p-m} \left(A + B \sin[c + d\,x]^k\right) dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_),x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k),x]] /;
FreeQ[{c,d,A,B,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] && IntegerQ[m-1/2] &&
  Not[IntegerQ[p]]
```

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- **Basis:** $\partial_z \left(\sqrt{\text{Sec}[z]}\ \sqrt{\text{Cos}[z]}\right) = 0$
- **Rule: If** $k^2 = 1 \bigwedge p - \frac{1}{2} \in \mathbb{Z} \bigwedge 2\,m \notin \mathbb{Z}$, **then**

$$\int \left(\sin[c + d\,x]^{-k}\right)^m \left(A \left(\sin[c + d\,x]^k\right)^p + B \left(\sin[c + d\,x]^k\right)^{p+1}\right) dx \longrightarrow$$
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```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_),x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^j)^(m-p)*(A+B*sin[c+d*x]^k),x]] /;
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```

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + C\left(\sin[c+d\,x]^k\right)^{p+2}\right) dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $j^2 = k^2 = 1 \wedge (j = k \vee p \in \mathbb{Z})$, **then**

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + C\left(\sin[c+d\,x]^k\right)^{p+2}\right) dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^j\right)^{m+j\,k\,p} \left(A + C\sin[c+d\,x]^{2\,k}\right) dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_.+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symbol] :=
  Int[(sin[c+d*x]^j)^(m+j*k*p)*(A+C*sin[c+d*x]^(2*k)),x] /;
FreeQ[{c,d,A,C,m,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[p+2-r] &&
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  Int[(sin[c+d*x]^k)^(p-m)*(A+C*sin[c+d*x]^(2*k)),x] /;
FreeQ[{c,d,A,C,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] && IntegerQ[m] &&
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- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z \left(\sqrt{\text{Sec}[z]}\ \sqrt{\text{Cos}[z]}\right) = 0$
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$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A \left(\sin[c+d\,x]^k\right)^p + C \left(\sin[c+d\,x]^k\right)^{p+2}\right) dx \longrightarrow$$
$$\sqrt{\text{Csc}[c+d\,x]}\ \sqrt{\text{Sin}[c+d\,x]} \int \left(\sin[c+d\,x]^k\right)^{p-m} \left(A + C \sin[c+d\,x]^{2\,k}\right) dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(p-m)*(A+C*sin[c+d*x]^(2*k)),x]] /;
FreeQ[{c,d,A,C,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] && IntegerQ[m-1/2] &&
  Not[IntegerQ[p]]
```

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```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^j)^(m-p)*(A+C*sin[c+d*x]^(2*k)),x]] /;
FreeQ[{c,d,A,C,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] && IntegerQ[p-1/2] &&
  Not[IntegerQ[2*m]]
```

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A+B\sin[c+d\,x]+C\sin[c+d\,x]^{-1}\right) dx$$

- **Derivation: Algebraic normalization**
- **Rule: If $j^2 = 1$, then**

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A+B\sin[c+d\,x]+C\sin[c+d\,x]^{-1}\right) dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^j\right)^{m-j} \left(C+A\sin[c+d\,x]+B\sin[c+d\,x]^2\right) dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.+B_.*sin[c_.+d_.*x_]+C_.*sin[c_.+d_.*x_]^(-1)),x_Symbol] :=
  Int[(sin[c+d*x]^j)^(m-j)*(C+A*sin[c+d*x]+B*sin[c+d*x]^2),x] /;
FreeQ[{c,d,A,B,C,m},x] && ZeroQ[j^2-1]
```

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1} + C\left(\sin[c+d\,x]^k\right)^{p+2}\right) dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $j^2 = k^2 = 1 \wedge (j = k \vee p \in \mathbb{Z})$, **then**

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$$\int \left(\sin[c+d\,x]^j\right)^{m+j\,k\,p} \left(A + B\sin[c+d\,x]^k + C\sin[c+d\,x]^{2\,k}\right) dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*
    (A_.*(sin[c_.+d_.*x_]^k_.)^p_.+B_.*(sin[c_.+d_.*x_]^k_.)^q_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symb
  Int[(sin[c+d*x]^j)^(m+j*k*p)*(A+B*sin[c+d*x]^k+C*sin[c+d*x]^(2*k)),x] /;
FreeQ[{c,d,A,B,C,m,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[p+1-q] && ZeroQ[p+2-r] &&
  (ZeroQ[j-k] || IntegerQ[p])
```

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- **Rule: If** $k^2 = 1 \wedge m \in \mathbb{Z} \wedge p \notin \mathbb{Z}$, **then**

$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1} + C\left(\sin[c+d\,x]^k\right)^{p+2}\right) dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^k\right)^{p-m} \left(A + B\sin[c+d\,x]^k + C\sin[c+d\,x]^{2\,k}\right) dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*
    (A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symbo
  Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k+C*sin[c+d*x]^(2*k)),x] /;
FreeQ[{c,d,A,B,C,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] && ZeroQ[p+2-r] &&
  IntegerQ[m] && Not[IntegerQ[p]]
```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z\left(\sqrt{\text{Sec}[z]}\,\sqrt{\text{Cos}[z]}\right)=0$
- **Rule: If** $k^2=1\bigwedge m-\frac{1}{2}\in\mathbb{Z}\bigwedge p\notin\mathbb{Z}$**, then**

$$\int\left(\sin[c+d\,x]^{-k}\right)^m\left(A\left(\sin[c+d\,x]^k\right)^p+B\left(\sin[c+d\,x]^k\right)^{p+1}+C\left(\sin[c+d\,x]^k\right)^{p+2}\right)dx\longrightarrow$$

$$\sqrt{\text{Csc}[c+d\,x]}\,\sqrt{\text{Sin}[c+d\,x]}\int\left(\sin[c+d\,x]^k\right)^{p-m}\left(A+B\sin[c+d\,x]^k+C\sin[c+d\,x]^{2\,k}\right)dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*
    (A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symbo
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k+C*sin[c+d*x]^(2*k)),x]] /;
FreeQ[{c,d,A,B,C,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] && ZeroQ[p+2-r] &&
  IntegerQ[m-1/2] && Not[IntegerQ[p]]
```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z\left(\sqrt{\text{Sec}[z]}\,\sqrt{\text{Cos}[z]}\right)=0$
- **Rule: If** $k^2=1\bigwedge p-\frac{1}{2}\in\mathbb{Z}\bigwedge 2\,m\notin\mathbb{Z}$**, then**

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$$\sqrt{\text{Csc}[c+d\,x]}\,\sqrt{\text{Sin}[c+d\,x]}\int\left(\sin[c+d\,x]^{-k}\right)^{m-p}\left(A+B\sin[c+d\,x]^k+C\sin[c+d\,x]^{2\,k}\right)dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*
    (A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symbo
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^j)^(m-p)*(A+B*sin[c+d*x]^k+C*sin[c+d*x]^(2*k)),x]] /;
FreeQ[{c,d,A,B,C,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] && ZeroQ[p+2-r] &&
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```

Integration Rules for $\int\left(A\left(\sin^i(z)\right)^p+B\left(\sin^i(z)\right)^{p+1}+C\left(\sin^i(z)\right)^{p+2}\right)\left(a+b\sin^k(z)\right)^n\,dz$ when $i^2=1\bigwedge k^2=1$

$$\int\left(A\left(\sin[c+d\,x]^k\right)^p+B\left(\sin[c+d\,x]^k\right)^{p+1}\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2=1\wedge p\neq 1$, **then**

$$\int\left(A\left(\sin[c+d\,x]^k\right)^p+B\left(\sin[c+d\,x]^k\right)^{p+1}\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx\longrightarrow$$
$$\int\left(\sin[c+d\,x]^k\right)^p\left(A+B\sin[c+d\,x]^k\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx$$

- **Program code:**

```
Int[(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_
  Int[(sin[c+d*x]^k)^p*(A+B*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,n,p},x] && ZeroQ[k^2-1] && ZeroQ[p+1-q] && Not[a===0 && b===1]
```

$$\int\left(A\left(\sin[c+d\,x]^{-k}\right)^p+B\left(\sin[c+d\,x]^{-k}\right)^{p+1}\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1$, **then**

$$\int\left(A+B\sin[c+d\,x]^{-k}\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx\longrightarrow$$
$$\int\sin[c+d\,x]^{-k}\left(B+A\sin[c+d\,x]^k\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx$$

- **Program code:**

```
Int[(A_+B_.*sin[c_.+d_.*x_]^i_.)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[sin[c+d*x]^(-k)*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,n},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && Not[a===0 && b===1]
```

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1 \land p \neq 1 \land p \neq -2$, **then**

$$\int\left(A\left(\sin[c+d\,x]^{-k}\right)^p+B\left(\sin[c+d\,x]^{-k}\right)^{p+1}\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx\longrightarrow$$
$$\int\left(\sin[c+d\,x]^{-k}\right)^{p+1}\left(B+A\sin[c+d\,x]^k\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx$$

- **Program code:**

```
Int[(A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_
  Int[(sin[c+d*x]^(-k))^(p+1)*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+1-q] &&
  Not[a===0 && b===1] && p≠-2
```

$$\int \left(A \left(\sin[c+d x]^k\right)^p + C \left(\sin[c+d x]^k\right)^{p+2}\right) \left(a+b \sin[c+d x]^k\right)^n \, dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1$**, then**

$$\int \left(A \left(\sin[c+d x]^k\right)^p + C \left(\sin[c+d x]^k\right)^{p+2}\right) \left(a+b \sin[c+d x]^k\right)^n \, dx \longrightarrow$$
$$\int \left(\sin[c+d x]^k\right)^p \left(A + C \sin[c+d x]^{2k}\right) \left(a+b \sin[c+d x]^k\right)^n \, dx$$

- **Program code:**

```
Int[(A_.*(sin[c_.+d_.*x_]^k_.)^p_.+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^k)^p*(A+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,n,p},x] && ZeroQ[k^2-1] && ZeroQ[p+2-r]
```

$$\int \left(A \left(\sin[c+d\,x]^{-k}\right)^{p} + C \left(\sin[c+d\,x]^{-k}\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^{k}\right)^{n} \, dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1$, **then**

$$\int \left(A + C\sin[c+d\,x]^{-2\,k}\right) \left(a+b\sin[c+d\,x]^{k}\right)^{n} \, dx \longrightarrow$$
$$\int \sin[c+d\,x]^{-2\,k} \left(C + A\sin[c+d\,x]^{2\,k}\right) \left(a+b\sin[c+d\,x]^{k}\right)^{n} \, dx$$

- **Program code:**

```
Int[(A_+C_.*sin[c_.+d_.*x_]^i2_)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[sin[c+d*x]^(-2*k)*(C+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,n},x] && ZeroQ[k^2-1] && ZeroQ[k+i2/2]
```

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1$, **then**

$$\int \left(A \left(\sin[c+d\,x]^{-k}\right)^{p} + C \left(\sin[c+d\,x]^{-k}\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^{k}\right)^{n} \, dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^{-k}\right)^{p+2} \left(C + A\sin[c+d\,x]^{2\,k}\right) \left(a+b\sin[c+d\,x]^{k}\right)^{n} \, dx$$

- **Program code:**

```
Int[(A_.*(sin[c_.+d_.*x_]^i_.)^p_.+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x
  Int[(sin[c+d*x]^(-k))^(p+2)*(C+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+2-r]
```

$$\int \left(A + B\sin[c+d\,x]^k + C\sin[c+d\,x]^{-k}\right)\left(a + b\sin[c+d\,x]^k\right)^n \,dx$$

- **Derivation: Algebraic normalization**
- **Rule: If $k^2 = 1$, then**

$$\int \left(A + B\sin[c+d\,x]^k + C\sin[c+d\,x]^{-k}\right)\left(a + b\sin[c+d\,x]^k\right)^n \,dx \longrightarrow$$
$$\int \sin[c+d\,x]^{-k}\left(C + A\sin[c+d\,x]^k + B\sin[c+d\,x]^{2k}\right)\left(a + b\sin[c+d\,x]^k\right)^n \,dx$$

- **Program code:**

```
Int[(A_.+B_.*sin[c_.+d_.*x_]^k_.+C_.*sin[c_.+d_.*x_]^l_.)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol
  Int[sin[c+d*x]^(-k)*(C+A*sin[c+d*x]^k+B*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,C,n},x] && ZeroQ[k^2-1] && ZeroQ[k+l]
```

$$\int \left(A \left(\sin[c+d\,x]^k\right)^p + B \left(\sin[c+d\,x]^k\right)^{p+1} + C \left(\sin[c+d\,x]^k\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n \, dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1$, **then**

$$\int \left(A \left(\sin[c+d\,x]^k\right)^p + B \left(\sin[c+d\,x]^k\right)^{p+1} + C \left(\sin[c+d\,x]^k\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n \, dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^k\right)^p \left(A + B\sin[c+d\,x]^k + C\sin[c+d\,x]^{2k}\right) \left(a+b\sin[c+d\,x]^k\right)^n \, dx$$

- **Program code:**

```
Int[(A_.*(sin[c_.+d_.*x_]^k_.)^p_.+B_.*(sin[c_.+d_.*x_]^k_.)^q_+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^k)^p*(A+B*sin[c+d*x]^k+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,C,n,p},x] && ZeroQ[k^2-1] && ZeroQ[p+1-q] && ZeroQ[p+2-r]
```

$$\int \left(A \left(\sin[c+d\,x]^{-k}\right)^p + B \left(\sin[c+d\,x]^{-k}\right)^{p+1} + C \left(\sin[c+d\,x]^{-k}\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n \, dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1$, **then**

$$\int \left(A + B\sin[c+d\,x]^{-k} + C\sin[c+d\,x]^{-2\,k}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx \longrightarrow$$
$$\int \sin[c+d\,x]^{-2\,k} \left(C + B\sin[c+d\,x]^k + A\sin[c+d\,x]^{2\,k}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx$$

- **Program code:**

```
Int[(A_.+B_.*sin[c_.+d_.*x_]^i_.+C_.*sin[c_.+d_.*x_]^i2_)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol
  Int[sin[c+d*x]^(-2*k)*(C+B*sin[c+d*x]^k+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,C,n},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[2*i-i2]
```

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1$, **then**

$$\int \left(A \left(\sin[c+d\,x]^{-k}\right)^p + B \left(\sin[c+d\,x]^{-k}\right)^{p+1} + C \left(\sin[c+d\,x]^{-k}\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^{-k}\right)^{p+2} \left(C + B\sin[c+d\,x]^k + A\sin[c+d\,x]^{2\,k}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx$$

- **Program code:**

```
Int[(A_.*(sin[c_.+d_.*x_]^i_.)^p_.+B_.*(sin[c_.+d_.*x_]^i_.)^q_+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^(-k))^(p+2)*(C+B*sin[c+d*x]^k+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,C,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+1-q] && ZeroQ[p+2-r]
```

Integration Rules for $\int(\sin^j(z))^m \left(A\left(\sin^i(z)\right)^p + B\left(\sin^i(z)\right)^{p+1} + C\left(\sin^i(z)\right)^{p+2}\right)\left(a+b\sin^k(z)\right)^n\, dz$ when $j^2 = 1 \bigwedge i^2 = 1 \bigwedge k^2 = 1$

$\int\left(\sin[c+d\,x]^j\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1}\right)\left(a+b\sin[c+d\,x]^k\right)^n dx$

- **Derivation: Algebraic normalization**
- **Rule: If** $j^2 = k^2 = 1 \wedge (j = k \vee p \in \mathbb{Z}) \wedge p \neq 1$, **then**

$$\int\left(\sin[c+d\,x]^j\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1}\right)\left(a+b\sin[c+d\,x]^k\right)^n dx \longrightarrow$$
$$\int\left(\sin[c+d\,x]^j\right)^{m+j\,k\,p}\left(A+B\sin[c+d\,x]^k\right)\left(a+b\sin[c+d\,x]^k\right)^n dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^j)^(m+j*k*p)*(A+B*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[p+1-q] &&
  (ZeroQ[j-k] || IntegerQ[p])
```

- **Derivation: Algebraic normalization**
- **Rule: If** $k^2 = 1 \wedge m \in \mathbb{Z} \wedge p \notin \mathbb{Z}$, **then**

$$\int\left(\sin[c+d\,x]^{-k}\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1}\right)\left(a+b\sin[c+d\,x]^k\right)^n dx \longrightarrow$$
$$\int\left(\sin[c+d\,x]^k\right)^{p-m}\left(A+B\sin[c+d\,x]^k\right)\left(a+b\sin[c+d\,x]^k\right)^n dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] &&
  IntegerQ[m] && Not[IntegerQ[p]]
```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z \left(\sqrt{\text{Sec}[z]}\ \sqrt{\text{Cos}[z]}\right) = 0$
- **Rule: If** $k^2 = 1 \bigwedge m - \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, **then**

$$\int \left(\sin[c + d\,x]^{-k}\right)^m \left(A \left(\sin[c + d\,x]^k\right)^p + B \left(\sin[c + d\,x]^k\right)^{p+1}\right) \left(a + b \sin[c + d\,x]^k\right)^n \, dx \longrightarrow$$
$$\sqrt{\text{Csc}[c + d\,x]}\ \sqrt{\text{Sin}[c + d\,x]} \int \left(\sin[c + d\,x]^k\right)^{p-m} \left(A + B \sin[c + d\,x]^k\right) \left(a + b \sin[c + d\,x]^k\right)^n \, dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x]] /;
FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] &&
  IntegerQ[m-1/2] && Not[IntegerQ[p]]
```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z \left(\sqrt{\text{Sec}[z]}\ \sqrt{\text{Cos}[z]}\right) = 0$
- **Rule: If** $k^2 = 1 \bigwedge p - \frac{1}{2} \in \mathbb{Z} \bigwedge 2\,m \notin \mathbb{Z}$, **then**

$$\int \left(\sin[c + d\,x]^{-k}\right)^m \left(A \left(\sin[c + d\,x]^k\right)^p + B \left(\sin[c + d\,x]^k\right)^{p+1}\right) \left(a + b \sin[c + d\,x]^k\right)^n \, dx \longrightarrow$$
$$\sqrt{\text{Csc}[c + d\,x]}\ \sqrt{\text{Sin}[c + d\,x]} \int \left(\sin[c + d\,x]^{-k}\right)^{m-p} \left(A + B \sin[c + d\,x]^k\right) \left(a + b \sin[c + d\,x]^k\right)^n \, dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^j)^(m-p)*(A+B*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x]] /;
FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] &&
  IntegerQ[p-1/2] && Not[IntegerQ[2*m]]
```

$$\int \left(\sin[c+d\,x]^{j}\right)^{m} \left(A\left(\sin[c+d\,x]^{-k}\right)^{p}+B\left(\sin[c+d\,x]^{-k}\right)^{p+1}\right)\left(a+b\sin[c+d\,x]^{k}\right)^{n}\,dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $j^2 = k^2 = 1$, **then**

$$\int \left(\sin[c+d\,x]^{j}\right)^{m}\left(A+B\sin[c+d\,x]^{-k}\right)\left(a+b\sin[c+d\,x]^{k}\right)^{n}\,dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^{j}\right)^{m-j\,k}\left(B+A\sin[c+d\,x]^{k}\right)\left(a+b\sin[c+d\,x]^{k}\right)^{n}\,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.+B_.*sin[c_.+d_.*x_]^i_.)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symb
  Int[(sin[c+d*x]^j)^(m-j*k)*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,m,n},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[k+i]
```

- **Derivation: Algebraic normalization**
- **Rule: If** $j^2 = k^2 = 1 \land (j+k=0 \lor p \in \mathbb{Z}) \land p \neq 1 \land p \neq -2$, **then**

$$\int \left(\sin[c+d\,x]^{j}\right)^{m}\left(A\left(\sin[c+d\,x]^{-k}\right)^{p}+B\left(\sin[c+d\,x]^{-k}\right)^{p+1}\right)\left(a+b\sin[c+d\,x]^{k}\right)^{n}\,dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^{j}\right)^{m-j\,k\,(p+1)}\left(B+A\sin[c+d\,x]^{k}\right)\left(a+b\sin[c+d\,x]^{k}\right)^{n}\,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^j)^(m-j*k*(p+1))*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[k+i] &&
  ZeroQ[p+1-q] && (ZeroQ[j+k] || IntegerQ[p]) && p≠-2
```

- **Derivation: Algebraic normalization**
- **Rule:** If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

$$\int \left(\sin[c+d\,x]^k\right)^m \left(A\,\left(\sin[c+d\,x]^{-k}\right)^p + B\,\left(\sin[c+d\,x]^{-k}\right)^{p+1}\right)\,\left(a+b\sin[c+d\,x]^k\right)^n\,dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^{-k}\right)^{p-m+1}\,\left(B+A\sin[c+d\,x]^k\right)\,\left(a+b\sin[c+d\,x]^k\right)^n\,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^(-k))^(p-m+1)*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+1-q] &&
  IntegerQ[m] && Not[IntegerQ[p]]
```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z \left(\sqrt{\text{Sec}[z]}\,\sqrt{\text{Cos}[z]}\right) = 0$
- **Rule:** If $k^2 = 1 \bigwedge m - \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

$$\int \left(\sin[c+d\,x]^k\right)^m \left(A\,\left(\sin[c+d\,x]^{-k}\right)^p + B\,\left(\sin[c+d\,x]^{-k}\right)^{p+1}\right)\,\left(a+b\sin[c+d\,x]^k\right)^n\,dx \longrightarrow$$
$$\sqrt{\text{Csc}[c+d\,x]}\,\sqrt{\text{Sin}[c+d\,x]}\,\int \left(\sin[c+d\,x]^{-k}\right)^{p-m+1}\,\left(B+A\sin[c+d\,x]^k\right)\,\left(a+b\sin[c+d\,x]^k\right)^n\,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^(-k))^(p-m+1)*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x]] /;
FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+1-q] &&
  IntegerQ[m-1/2] && Not[IntegerQ[p]]
```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z \left(\sqrt{\text{Sec}[z]}\ \sqrt{\text{Cos}[z]}\right) = 0$
- **Rule: If** $k^2 = 1\ \bigwedge\ p - \frac{1}{2} \in \mathbb{Z}\ \bigwedge\ 2\,m \notin \mathbb{Z}$**, then**

$$\int \left(\sin[c+d\,x]^k\right)^m \left(A \left(\sin[c+d\,x]^{-k}\right)^p + B \left(\sin[c+d\,x]^{-k}\right)^{p+1}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx \longrightarrow$$
$$\sqrt{\text{Csc}[c+d\,x]}\ \sqrt{\text{Sin}[c+d\,x]}\ \int \left(\sin[c+d\,x]^k\right)^{m-p-1} \left(B+A\sin[c+d\,x]^k\right) \left(a+b\sin[c+d\,x]^k\right)^n dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(m-p-1)*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x]] /;
FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+1-q] &&
  IntegerQ[p-1/2] && Not[IntegerQ[2*m]]
```

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A \left(\sin[c+d\,x]^k\right)^p + C \left(\sin[c+d\,x]^k\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n \,dx$$

- **Derivation: Algebraic normalization**
- **Rule: If $j^2 = k^2 = 1 \land (j = k \lor p \in \mathbb{Z})$, then**

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A \left(\sin[c+d\,x]^k\right)^p + C \left(\sin[c+d\,x]^k\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n \,dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^j\right)^{m+j\,k\,p} \left(A + C\sin[c+d\,x]^{2\,k}\right) \left(a+b\sin[c+d\,x]^k\right)^n \,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_.+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^j)^(m+j*k*p)*(A+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[p+2-r] &&
  (ZeroQ[j-k] || IntegerQ[p])
```

- **Derivation: Algebraic normalization**
- **Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then**

$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A \left(\sin[c+d\,x]^k\right)^p + C \left(\sin[c+d\,x]^k\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n \,dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^k\right)^{p-m} \left(A + C\sin[c+d\,x]^{2\,k}\right) \left(a+b\sin[c+d\,x]^k\right)^n \,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^k)^(p-m)*(A+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] &&
  IntegerQ[m] && Not[IntegerQ[p]]
```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z \left(\sqrt{\text{Sec}[z]}\ \sqrt{\text{Cos}[z]}\right) = 0$
- **Rule: If** $k^2 = 1 \bigwedge m - \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, **then**

$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A \left(\sin[c+d\,x]^k\right)^p + C \left(\sin[c+d\,x]^k\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n \,dx \longrightarrow$$
$$\sqrt{\text{Csc}[c+d\,x]}\ \sqrt{\text{Sin}[c+d\,x]} \int \left(\sin[c+d\,x]^k\right)^{p-m} \left(A + C\sin[c+d\,x]^{2k}\right) \left(a+b\sin[c+d\,x]^k\right)^n \,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(p-m)*(A+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x]] /;
FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] &&
  IntegerQ[m-1/2] && Not[IntegerQ[p]]
```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z \left(\sqrt{\text{Sec}[z]}\ \sqrt{\text{Cos}[z]}\right) = 0$
- **Rule: If** $k^2 = 1 \bigwedge p - \frac{1}{2} \in \mathbb{Z} \bigwedge 2\,m \notin \mathbb{Z}$, **then**

$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A \left(\sin[c+d\,x]^k\right)^p + C \left(\sin[c+d\,x]^k\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n \,dx \longrightarrow$$
$$\sqrt{\text{Csc}[c+d\,x]}\ \sqrt{\text{Sin}[c+d\,x]} \int \left(\sin[c+d\,x]^{-k}\right)^{m-p} \left(A + C\sin[c+d\,x]^{2k}\right) \left(a+b\sin[c+d\,x]^k\right)^n \,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^j)^(m-p)*(A+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x]] /;
FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] &&
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```

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A\left(\sin[c+d\,x]^{-k}\right)^p + C\left(\sin[c+d\,x]^{-k}\right)^{p+2}\right)\left(a+b\sin[c+d\,x]^k\right)^n \,dx$$

- **Derivation: Algebraic normalization**
- **Rule: If** $j^2 = k^2 = 1$, **then**

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A + C\sin[c+d\,x]^{-2k}\right)\left(a+b\sin[c+d\,x]^k\right)^n \,dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^j\right)^{m-2jk} \left(C + A\sin[c+d\,x]^{2k}\right)\left(a+b\sin[c+d\,x]^k\right)^n \,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.+C_.*sin[c_.+d_.*x_]^i2_)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symb
  Int[(sin[c+d*x]^j)^(m-2*j*k)*(C+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,m,n},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[k+i2/2]
```

- **Derivation: Algebraic normalization**
- **Rule: If** $j^2 = k^2 = 1 \land (j + k = 0 \lor p \in \mathbb{Z})$, **then**

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A\left(\sin[c+d\,x]^{-k}\right)^p + C\left(\sin[c+d\,x]^{-k}\right)^{p+2}\right)\left(a+b\sin[c+d\,x]^k\right)^n \,dx \longrightarrow$$
$$\int \left(\sin[c+d\,x]^j\right)^{m-jk(p+2)} \left(C + A\sin[c+d\,x]^{2k}\right)\left(a+b\sin[c+d\,x]^k\right)^n \,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_.+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Int[(sin[c+d*x]^j)^(m-j*k*(p+2))*(C+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[k+i] &&
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```

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```
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```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*
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```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.+B_.*sin[c_.+d_.*x_]^k_.+C_.*sin[c_.+d_.*x_]^l_.)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
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  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x]] /;
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```

$$\int \left(\sin[c+d\,x]^j\right)^m \left(A\,\left(\sin[c+d\,x]^{-k}\right)^p+B\,\left(\sin[c+d\,x]^{-k}\right)^{p+1}+C\,\left(\sin[c+d\,x]^{-k}\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n\,dx$$

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```
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```

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```

- **Derivation: Piecewise constant extraction and algebraic normalization**
- **Basis:** $\partial_z\left(\sqrt{\mathrm{Sec}[z]}\,\sqrt{\mathrm{Cos}[z]}\right)=0$
- **Rule: If** $k^2=1\bigwedge p-\frac{1}{2}\in\mathbb{Z}\bigwedge 2\,m\notin\mathbb{Z}$**, then**

$$\int\left(\sin[c+d\,x]^k\right)^m\left(A\left(\sin[c+d\,x]^{-k}\right)^p+B\left(\sin[c+d\,x]^{-k}\right)^{p+1}+C\left(\sin[c+d\,x]^{-k}\right)^{p+2}\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx\longrightarrow$$

$$\sqrt{\mathrm{Csc}[c+d\,x]}\,\sqrt{\mathrm{Sin}[c+d\,x]}\int\left(\sin[c+d\,x]^k\right)^{m-p-2}\left(C+B\sin[c+d\,x]^k+A\sin[c+d\,x]^{2k}\right)\left(a+b\sin[c+d\,x]^k\right)^n\,dx$$

- **Program code:**

```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*
    (A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
  Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(m-p-2)*(C+B*sin[c+d*x]^k+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x]] /;
FreeQ[{a,b,c,d,A,B,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+1-q] &&
  ZeroQ[p+2-r] && IntegerQ[p-1/2] && Not[IntegerQ[2*m]]
```

```
If[ShowSteps,

Int[u_,x_Symbol] :=
  Int[SubstInertSineForTrigOfLinear[u,x],x] /;
SimplifyFlag && FunctionOfTrigOfLinearQ[u,x],

Int[u_,x_Symbol] :=
  Int[SubstInertSineForTrigOfLinear[u,x],x] /;
FunctionOfTrigOfLinearQ[u,x]]
```

```
Int[u_,x_Symbol] :=
  Defer[Int[u,x]] /;
RecognizedFunctionOfTrigQ[u,x]
```