$$\int f[\mathrm{Sinh}[u]]\,\partial_x \mathrm{Sinh}[u]\,dx$$

- **Derivation: Integration by substitution**
- **Basis:** $f[\mathrm{Sinh}[z]]\,\mathrm{Cosh}[z] = f[\mathrm{Sinh}[z]]\,\partial_z \mathrm{Sinh}[z]$
- **Rule:**

$$\int f[\mathrm{Sinh}[a+b\,x]]\,\mathrm{Cosh}[a+b\,x]\,dx \longrightarrow \frac{1}{b}\,\mathrm{Subst}\Big[\int f[x]\,dx,\ x,\ \mathrm{Sinh}[a+b\,x]\Big]$$

- **Program code:**

```
Int[u_*Cosh[c_.*(a_.+b_.*x_)],x_Symbol] :=
  Dist[1/(b*c),Subst[Int[Regularize[SubstFor[Sinh[c*(a+b*x)],u,x],x],x],x,Sinh[c*(a+b*x)]]] /;
FreeQ[{a,b,c},x] && FunctionOfQ[Sinh[c*(a+b*x)],u,x,True]
```

- **Derivation: Integration by substitution**
- **Basis:** $f[\mathrm{Sinh}[z]]\,\mathrm{Coth}[z] = \frac{f[\mathrm{Sinh}[z]]}{\mathrm{Sinh}[z]}\,\partial_z \mathrm{Sinh}[z]$
- **Rule:**

$$\int f[\mathrm{Sinh}[a+b\,x]]\,\mathrm{Coth}[a+b\,x]\,dx \longrightarrow \frac{1}{b}\,\mathrm{Subst}\Big[\int \frac{f[x]}{x}\,dx,\ x,\ \mathrm{Sinh}[a+b\,x]\Big]$$

- **Program code:**

```
Int[u_*Coth[c_.*(a_.+b_.*x_)],x_Symbol] :=
  Dist[1/(b*c),Subst[Int[Regularize[SubstFor[Sinh[c*(a+b*x)],u,x]/x,x],x],x,Sinh[c*(a+b*x)]]] /;
FreeQ[{a,b,c},x] && FunctionOfQ[Sinh[c*(a+b*x)],u,x,True]
```

$$\int f[\text{Cosh}[u]]\, \partial_x \text{Cosh}[u]\, dx$$

- **Derivation: Integration by substitution**
- **Basis:** $f[\text{Cosh}[z]]\, \text{Sinh}[z] = f[\text{Cosh}[z]]\, \partial_z \text{Cosh}[z]$
- **Rule:**

$$\int f[\text{Cosh}[a+b\,x]]\, \text{Sinh}[a+b\,x]\, dx \longrightarrow \frac{1}{b}\, \text{Subst}\left[\int f[x]\, dx,\, x,\, \text{Cosh}[a+b\,x]\right]$$

- **Program code:**

```
Int[u_*Sinh[c_.*(a_.+b_.*x_)],x_Symbol] :=
  Dist[1/(b*c),Subst[Int[Regularize[SubstFor[Cosh[c*(a+b*x)],u,x],x],x],x,Cosh[c*(a+b*x)]]] /;
FreeQ[{a,b,c},x] && FunctionOfQ[Cosh[c*(a+b*x)],u,x,True]
```

- **Derivation: Integration by substitution**
- **Basis:** $f[\text{Cosh}[z]]\, \text{Tanh}[z] = \frac{f[\text{Cosh}[z]]}{\text{Cosh}[z]}\, \partial_z \text{Cosh}[z]$
- **Rule:**

$$\int f[\text{Cosh}[a+b\,x]]\, \text{Tanh}[a+b\,x]\, dx \longrightarrow \frac{1}{b}\, \text{Subst}\left[\int \frac{f[x]}{x}\, dx,\, x,\, \text{Cosh}[a+b\,x]\right]$$

- **Program code:**

```
Int[u_*Tanh[c_.*(a_.+b_.*x_)],x_Symbol] :=
  Dist[1/(b*c),Subst[Int[Regularize[SubstFor[Cosh[c*(a+b*x)],u,x]/x,x],x],x,Cosh[c*(a+b*x)]]] /;
FreeQ[{a,b,c},x] && FunctionOfQ[Cosh[c*(a+b*x)],u,x,True]
```

$$\int f[\mathrm{Coth}[u]]\,\partial_x \mathrm{Coth}[u]\,dx$$

- **Derivation: Integration by substitution**
- **Basis:** $f[\mathrm{Coth}[z]]\,\mathrm{Csch}[z]^2 = -f[\mathrm{Coth}[z]]\,\partial_z \mathrm{Coth}[z]$
- **Rule:**

$$\int f[\mathrm{Coth}[a+b\,x]]\,\mathrm{Csch}[a+b\,x]^2\,dx \longrightarrow -\frac{1}{b}\,\mathrm{Subst}\Big[\int f[x]\,dx,\,x,\,\mathrm{Coth}[a+b\,x]\Big]$$

- **Program code:**

```
Int[u_*Csch[c_.*(a_.+b_.*x_)]^2,x_Symbol] :=
  -Dist[1/(b*c),Subst[Int[Regularize[SubstFor[Coth[c*(a+b*x)],u,x],x],x],x,Coth[c*(a+b*x)]]] /;
FreeQ[{a,b,c},x] && FunctionOfQ[Coth[c*(a+b*x)],u,x,True] && NonsumQ[u]
```

- **Derivation: Integration by substitution**
- **Basis:** If $n \in \mathbb{Z}$, then $f[\mathrm{Coth}[z]]\,\mathrm{Tanh}[z]^n = \frac{f[\mathrm{Coth}[z]]}{\mathrm{Coth}[z]^n\,(1-\mathrm{Coth}[z]^2)}\,\partial_z \mathrm{Coth}[z]$
- **Rule:** If $n \in \mathbb{Z}$, then

$$\int f[\mathrm{Coth}[a+b\,x]]\,\mathrm{Tanh}[a+b\,x]^n\,dx \longrightarrow \frac{1}{b}\,\mathrm{Subst}\Big[\int \frac{f[x]}{x^n\,(1-x^2)}\,dx,\,x,\,\mathrm{Coth}[a+b\,x]\Big]$$

- **Program code:**

```
Int[u_*Tanh[c_.*(a_.+b_.*x_)]^n_.,x_Symbol] :=
  Dist[1/(b*c),Subst[Int[Regularize[SubstFor[Coth[c*(a+b*x)],u,x]/(x^n*(1-x^2)),x],x],x,Coth[c*(a+b*
FreeQ[{a,b,c},x] && IntegerQ[n] && FunctionOfQ[Coth[c*(a+b*x)],u,x,True] && TryPureTanhSubst[u*Tanh[
```

- **Derivation: Integration by substitution**

- **Basis:** $f[\text{Coth}[z]] = \frac{f[\text{Coth}[z]]}{1-\text{Coth}[z]^2}\,\partial_z \text{Coth}[z]$

- **Rule:**

$$\int f[\text{Coth}[a+b\,x]]\,dx \longrightarrow \frac{1}{b}\,\text{Subst}\left[\int \frac{f[x]}{1-x^2}\,dx,\ x,\ \text{Coth}[a+b\,x]\right]$$

- **Program code:**

```
If[ShowSteps,

Int[u_,x_Symbol] :=
  Module[{v=FunctionOfHyperbolic[u,x]},
  ShowStep["","Int[f[Coth[a+b*x]],x]","Subst[Int[f[x]/(1-x^2),x],x,Coth[a+b*x]]/b",Hold[
  Dist[1/Coefficient[v,x,1],Subst[Int[Regularize[SubstFor[Coth[v],u,x]/(1-x^2),x],x],x,Coth[v]]]]] /;
 NotFalseQ[v] && FunctionOfQ[Coth[v],u,x,True] && TryPureTanhSubst[u,x]] /;
SimplifyFlag,

Int[u_,x_Symbol] :=
  Module[{v=FunctionOfHyperbolic[u,x]},
  Dist[1/Coefficient[v,x,1],Subst[Int[Regularize[SubstFor[Coth[v],u,x]/(1-x^2),x],x],x,Coth[v]]] /;
 NotFalseQ[v] && FunctionOfQ[Coth[v],u,x,True] && TryPureTanhSubst[u,x]]]
```

$$\int f[\mathrm{Tanh}[u]]\,\partial_x \mathrm{Tanh}[u]\,dx$$

- **Derivation: Integration by substitution**
- **Basis:** $f[\mathrm{Tanh}[z]]\,\mathrm{Sech}[z]^2 = f[\mathrm{Tanh}[z]]\,\partial_z \mathrm{Tanh}[z]$
- **Rule:**

$$\int f[\mathrm{Tanh}[a+b\,x]]\,\mathrm{Sech}[a+b\,x]^2\,dx \longrightarrow \frac{1}{b}\,\mathrm{Subst}\left[\int f[x]\,dx,\,x,\,\mathrm{Tanh}[a+b\,x]\right]$$

- **Program code:**

```
Int[u_*Sech[c_.*(a_.+b_.*x_)]^2,x_Symbol] :=
  Dist[1/(b*c),Subst[Int[Regularize[SubstFor[Tanh[c*(a+b*x)],u,x],x],x],x,Tanh[c*(a+b*x)]]] /;
FreeQ[{a,b,c},x] && FunctionOfQ[Tanh[c*(a+b*x)],u,x,True] && NonsumQ[u]
```

- **Derivation: Integration by substitution**
- **Basis:** If $n\in\mathbb{Z}$, then $f[\mathrm{Tanh}[z]]\,\mathrm{Coth}[z]^n = \frac{f[\mathrm{Tanh}[z]]}{\mathrm{Tanh}[z]^n\,(1-\mathrm{Tanh}[z]^2)}\,\partial_z \mathrm{Tanh}[z]$
- **Rule:** If $n\in\mathbb{Z}$, then

$$\int f[\mathrm{Tanh}[a+b\,x]]\,\mathrm{Coth}[a+b\,x]^n\,dx \longrightarrow \frac{1}{b}\,\mathrm{Subst}\left[\int \frac{f[x]}{x^n\,(1-x^2)}\,dx,\,x,\,\mathrm{Tanh}[a+b\,x]\right]$$

- **Program code:**

```
Int[u_*Coth[c_.*(a_.+b_.*x_)]^n_.,x_Symbol] :=
  Dist[1/(b*c),Subst[Int[Regularize[SubstFor[Tanh[c*(a+b*x)],u,x]/(x^n*(1-x^2)),x],x],x,Tanh[c*(a+b*
FreeQ[{a,b,c},x] && IntegerQ[n] && FunctionOfQ[Tanh[c*(a+b*x)],u,x,True] && TryPureTanhSubst[u*Coth[
```

- **Derivation: Integration by substitution**
- **Basis:** $f[\text{Tanh}[z]] = \frac{f[\text{Tanh}[z]]}{1-\text{Tanh}[z]^2} \partial_z \text{Tanh}[z]$
- **Rule:**

$$\int f[\text{Tanh}[a+b\,x]]\,dx \longrightarrow \frac{1}{b}\,\text{Subst}\left[\int \frac{f[x]}{1-x^2}\,dx,\,x,\,\text{Tanh}[a+b\,x]\right]$$

- **Program code:**

```
If[ShowSteps,

Int[u_,x_Symbol] :=
  Module[{v=FunctionOfHyperbolic[u,x]},
  ShowStep["","Int[f[Tanh[a+b*x]],x]","Subst[Int[f[x]/(1-x^2),x],x,Tanh[a+b*x]]/b",Hold[
  Dist[1/Coefficient[v,x,1],Subst[Int[Regularize[SubstFor[Tanh[v],u,x]/(1-x^2),x],x],x,Tanh[v]]]]] /;
 NotFalseQ[v] && FunctionOfQ[Tanh[v],u,x,True] && TryPureTanhSubst[u,x]] /;
SimplifyFlag,

Int[u_,x_Symbol] :=
  Module[{v=FunctionOfHyperbolic[u,x]},
  Dist[1/Coefficient[v,x,1],Subst[Int[Regularize[SubstFor[Tanh[v],u,x]/(1-x^2),x],x],x,Tanh[v]]] /;
 NotFalseQ[v] && FunctionOfQ[Tanh[v],u,x,True] && TryPureTanhSubst[u,x]]]
```

$\int \text{TrigSimplify}[u]\, dx$

- **Derivation: Algebraic simplification**
- **Note: TrigSimplify needs to be tried after trig and hyperbolic rules are tried, but before unrestricted trig and hyperbolic substitutions!**
- **Rule: If trig simplification simplifes u, then**

$$\int u\, dx \longrightarrow \int \text{TrigSimplify}[u]\, dx$$

- **Program code:**

```
Int[u_,x_Symbol] :=
  Module[{v=TrigSimplify[u]},
  Int[v,x] /;
 v=!=u] /;
Not[MatchQ[u,w_.*(a_.+b_.*v_)^m_.*(c_.+d_.*v_)^n_. /;
        FreeQ[{a,b,c,d},x] && IntegersQ[m,n] && m<0 && n<0]]
```