Special Function Integration Problem 1

$$\int x^m \text{Gamma}[n, a + b x] \, dx$$

- *Rubi* uses reduction rules to integrate all the expressions:

```
Int[Gamma[n, a + b x], x]
```

$$\frac{(a + b x) \text{Gamma}[n, a + b x]}{b} - \frac{\text{Gamma}[1 + n, a + b x]}{b}$$

```
Int[x Gamma[n, a + b x], x]
```

$$-\frac{1}{2} \left(\frac{a^2}{b^2} - x^2\right) \text{Gamma}[n, a + b x] + \frac{a \text{Gamma}[1 + n, a + b x]}{b^2} - \frac{\text{Gamma}[2 + n, a + b x]}{2 b^2}$$

```
Int[x^2 Gamma[n, a + b x], x]
```

$$\frac{1}{3} \left(\frac{a^3}{b^3} + x^3\right) \text{Gamma}[n, a + b x] - \frac{a^2 \text{Gamma}[1 + n, a + b x]}{b^3} + \frac{a \text{Gamma}[2 + n, a + b x]}{b^3} - \frac{\text{Gamma}[3 + n, a + b x]}{3 b^3}$$

- *Mathematica* is able to integrate all the expressions:

$$\int \text{Gamma}[n, a + b x] \, dx$$

$$\frac{a \text{Gamma}[n, a + b x]}{b} + x \text{Gamma}[n, a + b x] - \frac{\text{Gamma}[1 + n, a + b x]}{b}$$

$$\int x \text{Gamma}[n, a + b x] \, dx$$

$$\frac{(-a^2 + b^2 x^2) \text{Gamma}[n, a + b x] + 2 a \text{Gamma}[1 + n, a + b x] - \text{Gamma}[2 + n, a + b x]}{2 b^2}$$

$$\int x^2 \text{Gamma}[n, a + b x] \, dx$$

$$\frac{1}{3 b^3} \left((a^3 + b^3 x^3) \text{Gamma}[n, a + b x] - 3 a^2 \text{Gamma}[1 + n, a + b x] + 3 a \text{Gamma}[2 + n, a + b x] - \text{Gamma}[3 + n, a + b x]\right)$$

- *Maple* is unable to integrate any of the expressions:

```
int (GAMMA (n, a + b * x), x);
```

$$\int \text{Gamma}[n, a + b x] \, dx$$

```
int (x * GAMMA (n, a + b * x), x);
```

$$\int x \, \mathrm{Gamma}[n, a + b x] \, dx$$

```
int(x^2*GAMMA(n, a+b*x), x);
```

$$\int x^2 \, \mathrm{Gamma}[n, a + b x] \, dx$$

Special Function Integration Problem 2

$$\int x^m \operatorname{PolyLog}\left[n, c\, e^{a+b x}\right] dx$$

- *Rubi* uses integration by parts to reduce the degree of x^m:

```
Int[PolyLog[n, c e^(a+b x)], x]
```

$$\frac{\operatorname{PolyLog}\left[1+n, c\, e^{a+b x}\right]}{b}$$

```
Int[x PolyLog[n, c e^(a+b x)], x]
```

$$\frac{x \operatorname{PolyLog}\left[1+n, c\, e^{a+b x}\right]}{b}-\frac{\operatorname{PolyLog}\left[2+n, c\, e^{a+b x}\right]}{b^2}$$

```
Int[x^2 PolyLog[n, c e^(a+b x)], x]
```

$$\frac{x^2 \operatorname{PolyLog}\left[1+n, c\, e^{a+b x}\right]}{b}-\frac{2 x \operatorname{PolyLog}\left[2+n, c\, e^{a+b x}\right]}{b^2}+\frac{2 \operatorname{PolyLog}\left[3+n, c\, e^{a+b x}\right]}{b^3}$$

- *Mathematica* does not know to use integration by parts:

$$\int \operatorname{PolyLog}\left[n, c\, e^{a+b x}\right] dx$$

$$\frac{\operatorname{PolyLog}\left[1+n, c\, e^{a+b x}\right]}{b}$$

$$\int x \operatorname{PolyLog}\left[n, c\, e^{a+b x}\right] dx$$

$$\int x \operatorname{PolyLog}\left[n, c\, e^{a+b x}\right] dx$$

$$\int x^2 \operatorname{PolyLog}\left[n, c\, e^{a+b x}\right] dx$$

$$\int x^2 \operatorname{PolyLog}\left[n, c\, e^{a+b x}\right] dx$$

- *Maple* does not know to use integration by parts:

```
int(polylog(n, c*exp(a+b*x)), x);
```

$$\int \operatorname{PolyLog}\left[n, c\, e^{a+b x}\right] dx$$

```
int(polylog(n, exp(a+b*x)), x);
```

$$\frac{\text{PolyLog}\left[n+1,\ e^{a+b\,x}\right]}{b}$$

```
int (x * polylog (n, exp (a + b * x)), x);
```

$$\int x\ \text{PolyLog}\left[n,\ e^{a+b\,x}\right]\,dx$$

```
int (x^2 * polylog (n, exp (a + b * x)), x);
```

$$\int x^2\ \text{PolyLog}\left[n,\ e^{a+b\,x}\right]\,dx$$

Special Function Integration Problem 3

$$\int \frac{\text{Log}[x]^m\,\text{PolyLog}[n,\,a\,x]}{x}\,dx$$

- *Rubi* uses integration by parts to reduce the degree of **Log[x]**^{m}:

$$\text{Int}\left[\frac{\text{Log}[x]\,\text{PolyLog}[n,\,a\,x]}{x},\,x\right]$$

$$\text{Log}[x]\,\text{PolyLog}[1+n,\,a\,x]-\text{PolyLog}[2+n,\,a\,x]$$

$$\text{Int}\left[\frac{\text{Log}[x]^2\,\text{PolyLog}[n,\,a\,x]}{x},\,x\right]$$

$$\text{Log}[x]^2\,\text{PolyLog}[1+n,\,a\,x]-2\,\text{Log}[x]\,\text{PolyLog}[2+n,\,a\,x]+2\,\text{PolyLog}[3+n,\,a\,x]$$

$$\text{Int}\left[\frac{\text{Log}[x]^3\,\text{PolyLog}[n,\,a\,x]}{x},\,x\right]$$

$$\text{Log}[x]^3\,\text{PolyLog}[1+n,\,a\,x]-3\,\text{Log}[x]^2\,\text{PolyLog}[2+n,\,a\,x]+6\,\text{Log}[x]\,\text{PolyLog}[3+n,\,a\,x]-6\,\text{PolyLog}[4+n,\,a\,x]$$

- *Mathematica* does not know to use integration by parts:

$$\int \frac{\text{Log}[x]\,\text{PolyLog}[n,\,a\,x]}{x}\,dx$$

$$\int \frac{\text{Log}[x]\,\text{PolyLog}[n,\,a\,x]}{x}\,dx$$

$$\int \frac{\text{Log}[x]^2\,\text{PolyLog}[n,\,a\,x]}{x}\,dx$$

$$\int \frac{\text{Log}[x]^2\,\text{PolyLog}[n,\,a\,x]}{x}\,dx$$

$$\int \frac{\text{Log}[x]^3\,\text{PolyLog}[n,\,a\,x]}{x}\,dx$$

$$\int \frac{\text{Log}[x]^3\,\text{PolyLog}[n,\,a\,x]}{x}\,dx$$

- *Maple* does not know to use integration by parts:

```
int(log(x)*polylog(n,a*x)/x,x);
```

$$\int \frac{\text{Log}[x]\,\text{PolyLog}[n,\,a\,x]}{x}\,dx$$

```
int(log(x)^2*polylog(n,a*x)/x,x);
```

$$\int \frac{\text{Log}[x]^2 \, \text{PolyLog}[n, a\,x]}{x} \, dx$$

```
int(log(x)^3*polylog(n, a*x)/x, x);
```

$$\int \frac{\text{Log}[x]^3 \, \text{PolyLog}[n, a\,x]}{x} \, dx$$

Special Function Integration Problem 4

$$\int x^m \, \text{ProductLog}[a + b\,x] \, dx$$

- *Rubi* uses integration by parts to reduce the degree of x^m:

```
Int[ProductLog[a + b x], x]
```

$$-x + \frac{a + b\,x}{b\,\text{ProductLog}[a + b\,x]} + \frac{(a + b\,x)\,\text{ProductLog}[a + b\,x]}{b}$$

```
Int[x ProductLog[a + b x], x]
```

$$\frac{a\,(a + b\,x)}{b^2} - \frac{(a + b\,x)^2}{4\,b^2} - \frac{(a + b\,x)^2}{8\,b^2\,\text{ProductLog}[a + b\,x]^2} - \frac{a\,(a + b\,x)}{b^2\,\text{ProductLog}[a + b\,x]} + \frac{(a + b\,x)^2}{4\,b^2\,\text{ProductLog}[a + b\,x]} - \frac{a\,(a + b\,x)\,\text{ProductLog}[a + b\,x]}{b^2} + \frac{(a + b\,x)^2\,\text{ProductLog}[a + b\,x]}{2\,b^2}$$

```
Int[x^2 ProductLog[a + b x], x]
```

$$-\frac{a^2\,(a + b\,x)}{b^3} + \frac{a\,(a + b\,x)^2}{2\,b^3} - \frac{(a + b\,x)^3}{9\,b^3} + \frac{2\,(a + b\,x)^3}{81\,b^3\,\text{ProductLog}[a + b\,x]^3} + \frac{a\,(a + b\,x)^2}{4\,b^3\,\text{ProductLog}[a + b\,x]^2} - \frac{2\,(a + b\,x)^3}{27\,b^3\,\text{ProductLog}[a + b\,x]^2} + \frac{a^2\,(a + b\,x)}{b^3\,\text{ProductLog}[a + b\,x]} - \frac{a\,(a + b\,x)^2}{2\,b^3\,\text{ProductLog}[a + b\,x]} + \frac{(a + b\,x)^3}{9\,b^3\,\text{ProductLog}[a + b\,x]} + \frac{a^2\,(a + b\,x)\,\text{ProductLog}[a + b\,x]}{b^3} - \frac{a\,(a + b\,x)^2\,\text{ProductLog}[a + b\,x]}{b^3} + \frac{(a + b\,x)^3\,\text{ProductLog}[a + b\,x]}{3\,b^3}$$

- *Mathematica* does not know to use integration by parts:

$$\int \text{ProductLog}[a + b\,x] \, dx$$

$$\frac{(a + b\,x)\,\left(1 - \text{ProductLog}[a + b\,x] + \text{ProductLog}[a + b\,x]^2\right)}{b\,\text{ProductLog}[a + b\,x]}$$

$$\int x\,\text{ProductLog}[a + b\,x] \, dx$$

$$\int x\,\text{ProductLog}[a + b\,x] \, dx$$

$$\int x^2\,\text{ProductLog}[a + b\,x] \, dx$$

$$\int x^2\,\text{ProductLog}[a + b\,x] \, dx$$

- *Maple* uses integration by parts to reduce the degree of x^m:

```
int (LambertW (a + b * x), x);
```

$$\frac{(a + b\,x)\left(1 - \mathrm{LambertW}(a + b\,x) + \mathrm{LambertW}(a + b\,x)^2\right)}{b\,\mathrm{LambertW}(a + b\,x)}$$

```
int (x * LambertW (a + b * x), x);
```

$$\frac{a\,(a + b\,x)}{b^2} - \frac{(a + b\,x)^2}{4\,b^2} - \frac{(a + b\,x)^2}{8\,b^2\,\mathrm{LambertW}(a + b\,x)^2} - \frac{a\,(a + b\,x)}{b^2\,\mathrm{LambertW}(a + b\,x)} + \frac{(a + b\,x)^2}{4\,b^2\,\mathrm{LambertW}(a + b\,x)} - \frac{a\,(a + b\,x)\,\mathrm{LambertW}(a + b\,x)}{b^2} + \frac{(a + b\,x)^2\,\mathrm{LambertW}(a + b\,x)}{2\,b^2}$$

```
int (x^2 * LambertW (a + b * x), x);
```

$$-\frac{a^2\,(a + b\,x)}{b^3} + \frac{a\,(a + b\,x)^2}{2\,b^3} - \frac{(a + b\,x)^3}{9\,b^3} + \frac{2\,(a + b\,x)^3}{81\,b^3\,\mathrm{LambertW}(a + b\,x)^3} + \frac{a\,(a + b\,x)^2}{4\,b^3\,\mathrm{LambertW}(a + b\,x)^2} - \frac{2\,(a + b\,x)^3}{27\,b^3\,\mathrm{LambertW}(a + b\,x)^2} + \frac{a^2\,(a + b\,x)}{b^3\,\mathrm{LambertW}(a + b\,x)} - \frac{a\,(a + b\,x)^2}{2\,b^3\,\mathrm{LambertW}(a + b\,x)} + \frac{(a + b\,x)^3}{9\,b^3\,\mathrm{LambertW}(a + b\,x)} + \frac{a^2\,(a + b\,x)\,\mathrm{LambertW}(a + b\,x)}{b^3} - \frac{a\,(a + b\,x)^2\,\mathrm{LambertW}(a + b\,x)}{b^3} + \frac{(a + b\,x)^3\,\mathrm{LambertW}(a + b\,x)}{3\,b^3}$$

Special Function Integration Problem 5

$$\int x^m \text{ProductLog}[a+b x]^2 \, dx$$

- *Rubi* uses integration by parts to reduce the degree of x^m:

```
Int[ProductLog[a + b x]^2, x]
```

$$4 x-\frac{4 (a+b x)}{b \text{ProductLog}[a+b x]}-\frac{2 (a+b x) \text{ProductLog}[a+b x]}{b}+\frac{(a+b x) \text{ProductLog}[a+b x]^2}{b}$$

```
Int[x ProductLog[a + b x]^2, x]
```

$$-\frac{4 a (a+b x)}{b^2}+\frac{3 (a+b x)^2}{4 b^2}+\frac{3 (a+b x)^2}{8 b^2 \text{ProductLog}[a+b x]^2}+\frac{4 a (a+b x)}{b^2 \text{ProductLog}[a+b x]}-\frac{3 (a+b x)^2}{4 b^2 \text{ProductLog}[a+b x]}+\frac{2 a (a+b x) \text{ProductLog}[a+b x]}{b^2}-\frac{(a+b x)^2 \text{ProductLog}[a+b x]}{2 b^2}-\frac{a (a+b x) \text{ProductLog}[a+b x]^2}{b^2}+\frac{(a+b x)^2 \text{ProductLog}[a+b x]^2}{2 b^2}$$

```
Int[x^2 ProductLog[a + b x]^2, x]
```

$$\frac{4 a^2 (a+b x)}{b^3}-\frac{3 a (a+b x)^2}{2 b^3}+\frac{8 (a+b x)^3}{27 b^3}-\frac{16 (a+b x)^3}{243 b^3 \text{ProductLog}[a+b x]^3}-\frac{3 a (a+b x)^2}{4 b^3 \text{ProductLog}[a+b x]^2}+\frac{16 (a+b x)^3}{81 b^3 \text{ProductLog}[a+b x]^2}-\frac{4 a^2 (a+b x)}{b^3 \text{ProductLog}[a+b x]}+\frac{3 a (a+b x)^2}{2 b^3 \text{ProductLog}[a+b x]}-\frac{8 (a+b x)^3}{27 b^3 \text{ProductLog}[a+b x]}-\frac{2 a^2 (a+b x) \text{ProductLog}[a+b x]}{b^3}+\frac{a (a+b x)^2 \text{ProductLog}[a+b x]}{b^3}-\frac{2 (a+b x)^3 \text{ProductLog}[a+b x]}{9 b^3}+\frac{a^2 (a+b x) \text{ProductLog}[a+b x]^2}{b^3}-\frac{a (a+b x)^2 \text{ProductLog}[a+b x]^2}{b^3}+\frac{(a+b x)^3 \text{ProductLog}[a+b x]^2}{3 b^3}$$

- *Mathematica* does not know to use integration by parts:

$$\int \text{ProductLog}[a+b x]^2 \, dx$$

$$\frac{(a+b x) \left(-4+4 \text{ProductLog}[a+b x]-2 \text{ProductLog}[a+b x]^2+\text{ProductLog}[a+b x]^3\right)}{b \text{ProductLog}[a+b x]}$$

$$\int x \text{ProductLog}[a+b x]^2 \, dx$$

$$\int x \text{ProductLog}[a+b x]^2 \, dx$$

$$\int x^2 \text{ProductLog}[a+b x]^2 \, dx$$

$$\int x^2 \, \text{ProductLog}[a + b x]^2 \, dx$$

- *Maple* uses integration by parts to reduce the degree of $\mathbf{x^m}$:

```
int(LambertW(a+b*x)^2, x);
```

$$\frac{\left(-4 + 4\,\text{LambertW}(a+b\,x) - 2\,\text{LambertW}(a+b\,x)^2 + \text{LambertW}(a+b\,x)^3\right)(a+b\,x)}{b\,\text{LambertW}(a+b\,x)}$$

```
int(x*LambertW(a+b*x)^2, x);
```

$$-\frac{4\,a\,(a+b\,x)}{b^2} + \frac{3\,(a+b\,x)^2}{4\,b^2} + \frac{3\,(a+b\,x)^2}{8\,b^2\,(\text{LambertW}(a+b\,x))^2} + \frac{4\,a\,(a+b\,x)}{b^2\,\text{LambertW}(a+b\,x)} - \frac{3\,(a+b\,x)^2}{4\,b^2\,\text{LambertW}(a+b\,x)} + \frac{2\,a\,(a+b\,x)\,\text{LambertW}(a+b\,x)}{b^2} - \frac{(a+b\,x)^2\,\text{LambertW}(a+b\,x)}{2\,b^2} - \frac{a\,(a+b\,x)\,(\text{LambertW}(a+b\,x))^2}{b^2} + \frac{(a+b\,x)^2\,(\text{LambertW}(a+b\,x))^2}{2\,b^2}$$

```
int(x^2*LambertW(a+b*x)^2, x);
```

$$\frac{4\,a^2\,(a+b\,x)}{b^3} - \frac{3\,a\,(a+b\,x)^2}{2\,b^3} + \frac{8\,(a+b\,x)^3}{27\,b^3} - \frac{16\,(a+b\,x)^3}{243\,b^3\,(\text{LambertW}(a+b\,x))^3} - \frac{3\,a\,(a+b\,x)^2}{4\,b^3\,(\text{LambertW}(a+b\,x))^2} + \frac{16\,(a+b\,x)^3}{81\,b^3\,(\text{LambertW}(a+b\,x))^2} - \frac{4\,a^2\,(a+b\,x)}{b^3\,\text{LambertW}(a+b\,x)} + \frac{3\,a\,(a+b\,x)^2}{2\,b^3\,\text{LambertW}(a+b\,x)} - \frac{8\,(a+b\,x)^3}{27\,b^3\,\text{LambertW}(a+b\,x)} - \frac{2\,a^2\,(a+b\,x)\,\text{LambertW}(a+b\,x)}{b^3} + \frac{a\,(a+b\,x)^2\,\text{LambertW}(a+b\,x)}{b^3} - \frac{2\,(a+b\,x)^3\,\text{LambertW}(a+b\,x)}{9\,b^3} + \frac{a^2\,(a+b\,x)\,(\text{LambertW}(a+b\,x))^2}{b^3} - \frac{a\,(a+b\,x)^2\,(\text{LambertW}(a+b\,x))^2}{b^3} + \frac{(a+b\,x)^3\,(\text{LambertW}(a+b\,x))^2}{3\,b^3}$$

Special Function Integration Problem 6

$$\int x^m \text{ProductLog}\left[a\, x^2\right] dx$$

- *Rubi* is able to integrate $x^m \text{ProductLog}\left[a\, x^2\right]$ for all odd m:

```
Int[x^3 ProductLog[a x^2], x]
```

$$-\frac{x^4}{8}-\frac{x^4}{16\, \text{ProductLog}\left[a\, x^2\right]^2}+\frac{x^4}{8\, \text{ProductLog}\left[a\, x^2\right]}+\frac{1}{4} x^4\, \text{ProductLog}\left[a\, x^2\right]$$

```
Int[x ProductLog[a x^2], x]
```

$$-\frac{x^2}{2}+\frac{x^2}{2\, \text{ProductLog}\left[a\, x^2\right]}+\frac{1}{2} x^2\, \text{ProductLog}\left[a\, x^2\right]$$

```
Int[ProductLog[a x^2]/x, x]
```

$$\frac{1}{2} \text{ProductLog}\left[a\, x^2\right]+\frac{1}{4} \text{ProductLog}\left[a\, x^2\right]^2$$

```
Int[ProductLog[a x^2]/x^3, x]
```

$$\frac{1}{2} a\, \text{ExpIntegralEi}\left[-\text{ProductLog}\left[a\, x^2\right]\right]-\frac{\text{ProductLog}\left[a\, x^2\right]}{2\, x^2}$$

```
Int[ProductLog[a x^2]/x^5, x]
```

$$-\frac{1}{2} a^2\, \text{ExpIntegralEi}\left[-2\, \text{ProductLog}\left[a\, x^2\right]\right]-\frac{\text{ProductLog}\left[a\, x^2\right]}{2\, x^4}$$

- *Mathematica* is only able to integrate $x^m \text{ProductLog}\left[a\, x^2\right]$ for odd m greater than -2:

$$\int x^3\, \text{ProductLog}\left[a\, x^2\right] dx$$

$$-\frac{x^4}{8}-\frac{x^4}{16\, \text{ProductLog}\left[a\, x^2\right]^2}+\frac{x^4}{8\, \text{ProductLog}\left[a\, x^2\right]}+\frac{1}{4} x^4\, \text{ProductLog}\left[a\, x^2\right]$$

$$\int x\, \text{ProductLog}\left[a\, x^2\right] dx$$

$$-\frac{x^2}{2}+\frac{x^2}{2\, \text{ProductLog}\left[a\, x^2\right]}+\frac{1}{2} x^2\, \text{ProductLog}\left[a\, x^2\right]$$

$$\int \frac{\text{ProductLog}\left[a\,x^2\right]}{x}\,dx$$

$$\frac{1}{2}\,\text{ProductLog}\left[a\,x^2\right] + \frac{1}{4}\,\text{ProductLog}\left[a\,x^2\right]^2$$

$$\int \frac{\text{ProductLog}\left[a\,x^2\right]}{x^3}\,dx$$

$$\int \frac{\text{ProductLog}\left[a\,x^2\right]}{x^3}\,dx$$

$$\int \frac{\text{ProductLog}\left[a\,x^2\right]}{x^5}\,dx$$

$$\int \frac{\text{ProductLog}\left[a\,x^2\right]}{x^5}\,dx$$

- *Maple* is only able to integrate $x^m\,\text{LambertW}\left(a\,x^2\right)$ for m equal 1 or -1:

```
int (x^3 * LambertW (a * x^2), x);
```

$$\int x^3\,\text{LambertW}\left(a\,x^2\right)\,dx$$

```
int (x * LambertW (a * x^2), x);
```

$$-\frac{x^2}{2} + \frac{x^2}{2\,\text{LambertW}\left(a\,x^2\right)} + \frac{1}{2}\,x^2\,\text{LambertW}\left(a\,x^2\right)$$

```
int (LambertW (a * x^2) / x, x);
```

$$\frac{1}{2}\,\text{LambertW}\left(a\,x^2\right) + \frac{1}{4}\,\text{LambertW}\left(a\,x^2\right)^2$$

```
int (LambertW (a * x^2) / x^3, x);
```

$$\int \frac{\text{LambertW}\left(a\,x^2\right)}{x^3}\,dx$$

```
int (LambertW (a * x^2) / x^5, x);
```

$$\int \frac{\text{LambertW}\left(a\,x^2\right)}{x^5}\,dx$$

Special Function Integration Problem 7

$$\int x^m \text{ProductLog}\left[\frac{a}{x}\right] dx$$

- *Rubi* is able to integrate $x^m \text{ProductLog}\left[\frac{a}{x}\right]$ for all integer m:

$$\text{Int}\left[x^2 \text{ProductLog}\left[\frac{a}{x}\right], x\right]$$

$$-\frac{3}{2} a^3 \text{ExpIntegralEi}\left[-3 \text{ProductLog}\left[\frac{a}{x}\right]\right] + \frac{1}{2} x^3 \text{ProductLog}\left[\frac{a}{x}\right] - \frac{1}{2} x^3 \text{ProductLog}\left[\frac{a}{x}\right]^2$$

$$\text{Int}\left[x \text{ProductLog}\left[\frac{a}{x}\right], x\right]$$

$$a^2 \text{ExpIntegralEi}\left[-2 \text{ProductLog}\left[\frac{a}{x}\right]\right] + x^2 \text{ProductLog}\left[\frac{a}{x}\right]$$

$$\text{Int}\left[\text{ProductLog}\left[\frac{a}{x}\right], x\right]$$

$$-a \text{ExpIntegralEi}\left[-\text{ProductLog}\left[\frac{a}{x}\right]\right] + x \text{ProductLog}\left[\frac{a}{x}\right]$$

$$\text{Int}\left[\frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x}, x\right]$$

$$-\text{ProductLog}\left[\frac{a}{x}\right] - \frac{1}{2} \text{ProductLog}\left[\frac{a}{x}\right]^2$$

$$\text{Int}\left[\frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x^2}, x\right]$$

$$\frac{1}{x} - \frac{1}{x \text{ProductLog}\left[\frac{a}{x}\right]} - \frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x}$$

$$\text{Int}\left[\frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x^3}, x\right]$$

$$\frac{1}{4 x^2} + \frac{1}{8 x^2 \text{ProductLog}\left[\frac{a}{x}\right]^2} - \frac{1}{4 x^2 \text{ProductLog}\left[\frac{a}{x}\right]} - \frac{\text{ProductLog}\left[\frac{a}{x}\right]}{2 x^2}$$

$$\text{Int}\left[\frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x^4}, x\right]$$

$$\frac{1}{9 x^3} - \frac{2}{81 x^3 \text{ProductLog}\left[\frac{a}{x}\right]^3} + \frac{2}{27 x^3 \text{ProductLog}\left[\frac{a}{x}\right]^2} - \frac{1}{9 x^3 \text{ProductLog}\left[\frac{a}{x}\right]} - \frac{\text{ProductLog}\left[\frac{a}{x}\right]}{3 x^3}$$

Mathematica is only able to integrate x^m **ProductLog**$\left[\frac{a}{x}\right]$ for m equal -1 or -2:

$$\int x^2\,\text{ProductLog}\left[\frac{a}{x}\right]dx$$

$$\int x^2\,\text{ProductLog}\left[\frac{a}{x}\right]dx$$

$$\int x\,\text{ProductLog}\left[\frac{a}{x}\right]dx$$

$$\int x\,\text{ProductLog}\left[\frac{a}{x}\right]dx$$

$$\int \text{ProductLog}\left[\frac{a}{x}\right]dx$$

$$\int \text{ProductLog}\left[\frac{a}{x}\right]dx$$

$$\int \frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x}dx$$

$$-\text{ProductLog}\left[\frac{a}{x}\right]-\frac{1}{2}\,\text{ProductLog}\left[\frac{a}{x}\right]^2$$

$$\int \frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x^2}dx$$

$$\frac{1}{x}-\frac{1}{x\,\text{ProductLog}\left[\frac{a}{x}\right]}-\frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x}$$

$$\int \frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x^3}dx$$

$$\int \frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x^3}dx$$

$$\int \frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x^4}dx$$

$$\int \frac{\text{ProductLog}\left[\frac{a}{x}\right]}{x^4}dx$$

- *Maple* is able to integrate x^m **LambertW** $\left(\frac{a}{x}\right)$ for all integer m:

```
int(x^2*LambertW(a/x), x);
```

$$\frac{3}{2}\,a^3\,\text{Ei}\left(1,\,3\,\text{LambertW}\left(\frac{a}{x}\right)\right)+\frac{1}{2}\,x^3\,\text{LambertW}\left(\frac{a}{x}\right)-\frac{1}{2}\,x^3\,\text{LambertW}\left(\frac{a}{x}\right)^2$$

```
int (x * LambertW (a / x), x);
```

$$-a^2 \operatorname{Ei}\left(1, 2 \operatorname{LambertW}\left(\frac{a}{x}\right)\right) + x^2 \operatorname{LambertW}\left(\frac{a}{x}\right)$$

```
int (LambertW (a / x), x);
```

$$a \operatorname{Ei}\left(1, \operatorname{LambertW}\left(\frac{a}{x}\right)\right) + x \operatorname{LambertW}\left(\frac{a}{x}\right)$$

```
int (LambertW (a / x) / x, x);
```

$$-\operatorname{LambertW}\left(\frac{a}{x}\right) - \frac{1}{2} \operatorname{LambertW}\left(\frac{a}{x}\right)^2$$

```
int (LambertW (a / x) / x^2, x);
```

$$\frac{1}{x} - \frac{1}{x \operatorname{LambertW}\left(\frac{a}{x}\right)} - \frac{\operatorname{LambertW}\left(\frac{a}{x}\right)}{x}$$

```
int (LambertW (a / x) / x^3, x);
```

$$\frac{1}{4 x^2} + \frac{1}{8 x^2 \operatorname{LambertW}\left(\frac{a}{x}\right)^2} - \frac{1}{4 x^2 \operatorname{LambertW}\left(\frac{a}{x}\right)} - \frac{\operatorname{LambertW}\left(\frac{a}{x}\right)}{2 x^2}$$

```
int (LambertW (a / x) / x^4, x);
```

$$\frac{1}{9 x^3} - \frac{2}{81 x^3 \operatorname{LambertW}\left(\frac{a}{x}\right)^3} + \frac{2}{27 x^3 \operatorname{LambertW}\left(\frac{a}{x}\right)^2} - \frac{1}{9 x^3 \operatorname{LambertW}\left(\frac{a}{x}\right)} - \frac{\operatorname{LambertW}\left(\frac{a}{x}\right)}{3 x^3}$$

Special Function Integration Problem 8

$$\int \text{ProductLog}\left[\frac{a}{x^{1/(n-1)}}\right]^n \, dx$$

- *Rubi* knows how to integrate $\text{ProductLog}\left[\frac{a}{x^{1/(n-1)}}\right]^n$ for symbolic and numeric n:

$$\text{Int}\left[\text{ProductLog}\left[\frac{a}{x^{1/(n-1)}}\right]^n, x\right]$$

$$\frac{n}{n-1} x \, \text{ProductLog}\left[\frac{a}{x^{1/(n-1)}}\right]^{n-1} + x \, \text{ProductLog}\left[\frac{a}{x^{1/(n-1)}}\right]^n$$

$$\text{Int}\left[\text{ProductLog}\left[\frac{a}{x}\right]^2, x\right]$$

$$2 x \, \text{ProductLog}\left[\frac{a}{x}\right] + x \, \text{ProductLog}\left[\frac{a}{x}\right]^2$$

$$\text{Int}\left[\text{ProductLog}\left[\frac{a}{x^{1/2}}\right]^3, x\right]$$

$$\frac{3}{2} x \, \text{ProductLog}\left[\frac{a}{\sqrt{x}}\right]^2 + x \, \text{ProductLog}\left[\frac{a}{\sqrt{x}}\right]^3$$

$$\text{Int}\left[\text{ProductLog}\left[\frac{a}{x^{1/3}}\right]^4, x\right]$$

$$\frac{4}{3} x \, \text{ProductLog}\left[\frac{a}{x^{1/3}}\right]^3 + x \, \text{ProductLog}\left[\frac{a}{x^{1/3}}\right]^4$$

- *Mathematica* does not know how to integrate $\text{ProductLog}\left[\frac{a}{x^{1/(n-1)}}\right]^n$ for symbolic or numeric n:

$$\int \text{ProductLog}\left[\frac{a}{x^{1/(n-1)}}\right]^n \, dx$$

$$\int \text{ProductLog}\left[\frac{a}{x^{1/(n-1)}}\right]^n \, dx$$

$$\int \text{ProductLog}\left[\frac{a}{x}\right]^2 \, dx$$

$$\int \text{ProductLog}\left[\frac{a}{x}\right]^2 \, dx$$

$$\int \text{ProductLog}\left[\frac{a}{x^{1/2}}\right]^3 \, dx$$

$$\int \text{ProductLog}\left[\frac{a}{\sqrt{x}}\right]^3 \, dx$$

```
∫ProductLog[a/x^(1/3)]^4 dx
```

$$\int \text{ProductLog}\left[\frac{a}{x^{1/3}}\right]^4 dx$$

- *Maple* does not know how to integrate **LambertW** $\left(\frac{a}{x^{1/(n-1)}}\right)^n$ for symbolic n:

```
int (LambertW (a / x^(1 / (n - 1)))^n, x);
```

$$\int \text{LambertW}\left(\frac{a}{x^{1/(n-1)}}\right)^n dx$$

```
int (LambertW (a / x)^2, x);
```

$$2\, x\, \text{LambertW}\left(\frac{a}{x}\right) + x\, \text{LambertW}\left(\frac{a}{x}\right)^2$$

```
int (LambertW (a / x^(1 / 2))^3, x);
```

$$\frac{3}{2}\, x\, \text{LambertW}\left(\frac{a}{\sqrt{x}}\right)^2 + x\, \text{LambertW}\left(\frac{a}{\sqrt{x}}\right)^3$$

```
int (LambertW (a / x^(1 / 3))^4, x);
```

$$\frac{4}{3}\, x\, \text{LambertW}\left(\frac{a}{x^{1/3}}\right)^3 + x\, \text{LambertW}\left(\frac{a}{x^{1/3}}\right)^4$$

Special Function Integration Problem 9

$$\int \frac{\text{ProductLog}[a\,x^n]^p}{x^{n\,(p-1)+1}}\,dx$$

- *Rubi* knows how to integrate $\frac{\text{ProductLog}[a\,x^n]^p}{x^{n\,(p-1)+1}}$ for symbolic and numeric n and p:

$$\text{Int}\left[\frac{\text{ProductLog}[a\,x^n]^p}{x^{n\,(p-1)+1}},\,x\right]$$

$$-\frac{p\,\text{ProductLog}[a\,x^n]^{-1+p}}{n\,(p-1)^2\,x^{n\,(p-1)}}-\frac{\text{ProductLog}[a\,x^n]^p}{n\,(p-1)\,x^{n\,(p-1)}}$$

$$\text{Int}\left[\text{ProductLog}\left[\frac{a}{x}\right]^2,\,x\right]$$

$$2\,x\,\text{ProductLog}\left[\frac{a}{x}\right]+x\,\text{ProductLog}\left[\frac{a}{x}\right]^2$$

$$\text{Int}\left[\text{ProductLog}\left[\frac{a}{x^{1/2}}\right]^3,\,x\right]$$

$$\frac{3}{2}\,x\,\text{ProductLog}\left[\frac{a}{\sqrt{x}}\right]^2+x\,\text{ProductLog}\left[\frac{a}{\sqrt{x}}\right]^3$$

$$\text{Int}\left[\text{ProductLog}\left[\frac{a}{x^{1/3}}\right]^4,\,x\right]$$

$$\frac{4}{3}\,x\,\text{ProductLog}\left[\frac{a}{x^{1/3}}\right]^3+x\,\text{ProductLog}\left[\frac{a}{x^{1/3}}\right]^4$$

- *Mathematica* does not know how to integrate $\frac{\text{ProductLog}[a\,x^n]^p}{x^{n\,(p-1)+1}}$ for symbolic or numeric n and p:

$$\int \frac{\text{ProductLog}[a\,x^n]^p}{x^{n\,(p-1)+1}}\,dx$$

$$\int \frac{\text{ProductLog}[a\,x^n]^p}{x^{n\,(p-1)+1}}\,dx$$

$$\int \frac{\text{ProductLog}\left[a\,x^2\right]^p}{x^{2\,(p-1)+1}}\,dx$$

$$\int \frac{\text{ProductLog}\left[a\,x^2\right]^p}{x^{2\,(p-1)+1}}\,dx$$

$$\int \frac{\text{ProductLog}[a\,x^n]^3}{x^{n\,(3-1)+1}}\,dx$$

$$\int \frac{\text{ProductLog}[a\,x^n]^3}{x^{2\,n+1}}\,dx$$

$$\int \frac{\text{ProductLog}\left[a\,x^2\right]^3}{x^{2\,(3-1)+1}}\,dx$$

$$\int \frac{\text{ProductLog}\left[a\,x^2\right]^3}{x^5}\,dx$$

- *Maple* does not know how to integrate $\frac{\text{LambertW}\,(a\,x^n)^p}{x^{n\,(p-1)+1}}$ for symbolic or numeric n and p:

```
int (LambertW (a * x^n) ^p / x^ (n * (p - 1) + 1), x);
```

$$\int \frac{\text{LambertW}\,(a\,x^n)^p}{x^{n\,(p-1)+1}}\,dx$$

```
int (LambertW (a * x^2) ^p / x^ (2 * (p - 1) + 1), x);
```

$$\int \frac{\text{LambertW}\left(a\,x^2\right)^p}{x^{2\,(p-1)+1}}\,dx$$

```
int (LambertW (a * x^n) ^3 / x^ (n * (3 - 1) + 1), x);
```

$$\int \frac{\text{LambertW}\,(a\,x^n)^3}{x^{2\,n+1}}\,dx$$

```
int (LambertW (a * x^2) ^3 / x^ (2 * (3 - 1) + 1), x);
```

$$\int \frac{\text{LambertW}\left(a\,x^2\right)^3}{x^5}\,dx$$