Rational Function Integration Problem 1

$$\int x^{n-1} (a + b x^n)^m \, dx$$

- *Rubi* knows and takes advantage of the general rule for arbitrary m and n:

```
Int[x^(n-1) (a + b x^n)^m, x]
```

$$\frac{(a + b x^n)^{1+m}}{b (1 + m) n}$$

```
Int[x^(n-1) (a + b x^n)^16, x]
```

$$\frac{(a + b x^n)^{17}}{17 b n}$$

```
Int[x^2 (a + b x^3)^16, x]
```

$$\frac{(a + b x^3)^{17}}{51 b}$$

- *Mathematica* knows the rule for arbitrary m and n, but needlessly expands the integrand when m is a positive integer:

$$\int x^{n-1} (a + b x^n)^m \, dx$$

$$\frac{(a + b x^n)^{1+m}}{b n + b m n}$$

$$\int x^{n-1} (a + b x^n)^{16} \, dx$$

$$\frac{1}{17 n} x^n \left(17 a^{16} + 136 a^{15} b x^n + 680 a^{14} b^2 x^{2 n} + 2380 a^{13} b^3 x^{3 n} + 6188 a^{12} b^4 x^{4 n} + 12376 a^{11} b^5 x^{5 n} + 19448 a^{10} b^6 x^{6 n} + 24310 a^9 b^7 x^{7 n} + 24310 a^8 b^8 x^{8 n} + 19448 a^7 b^9 x^{9 n} + 12376 a^6 b^{10} x^{10 n} + 6188 a^5 b^{11} x^{11 n} + 2380 a^4 b^{12} x^{12 n} + 680 a^3 b^{13} x^{13 n} + 136 a^2 b^{14} x^{14 n} + 17 a b^{15} x^{15 n} + b^{16} x^{16 n}\right)$$

$$\int x^2 (a + b x^3)^{16} \, dx$$

$$\frac{a^{16} x^3}{3} + \frac{8}{3} a^{15} b x^6 + \frac{40}{3} a^{14} b^2 x^9 + \frac{140}{3} a^{13} b^3 x^{12} + \frac{364}{3} a^{12} b^4 x^{15} + \frac{728}{3} a^{11} b^5 x^{18} + \frac{1144}{3} a^{10} b^6 x^{21} + \frac{1430}{3} a^9 b^7 x^{24} + \frac{1430}{3} a^8 b^8 x^{27} + \frac{1144}{3} a^7 b^9 x^{30} + \frac{728}{3} a^6 b^{10} x^{33} + \frac{364}{3} a^5 b^{11} x^{36} + \frac{140}{3} a^4 b^{12} x^{39} + \frac{40}{3} a^3 b^{13} x^{42} + \frac{8}{3} a^2 b^{14} x^{45} + \frac{1}{3} a b^{15} x^{48} + \frac{b^{16} x^{51}}{51}$$

- *Maple* knows the rule for arbitrary m and n, but needlessly expands the integrand when m is a positive integer:

```
int(x^(n-1)*(a+b*x^n)^m, x);
```

$$\frac{(a+b\,x^n)\,(a+b\,x^n)^m}{b\,(1+m)\,n}$$

```
int(x^(n-1)*(a+b*x^n)^16, x);
```

$$\frac{1}{17\,n}\left(17\,a^{16}\,x^{n}+136\,a^{15}\,b\,x^{2\,n}+680\,a^{14}\,b^{2}\,x^{3\,n}+2380\,a^{13}\,b^{3}\,x^{4\,n}+6188\,a^{12}\,b^{4}\,x^{5\,n}+12\,376\,a^{11}\,b^{5}\,x^{6\,n}+19\,448\,a^{10}\,b^{6}\,x^{7\,n}+24\,310\,a^{9}\,b^{7}\,x^{8\,n}+24\,310\,a^{8}\,b^{8}\,x^{9\,n}+19\,448\,a^{7}\,b^{9}\,x^{10\,n}+12\,376\,a^{6}\,b^{10}\,x^{11\,n}+6188\,a^{5}\,b^{11}\,x^{12\,n}+2380\,a^{4}\,b^{12}\,x^{13\,n}+680\,a^{3}\,b^{13}\,x^{14\,n}+136\,a^{2}\,b^{14}\,x^{15\,n}+17\,a\,b^{15}\,x^{16\,n}+b^{16}\,x^{17\,n}\right)$$

```
int(x^2*(a+b*x^3)^16, x);
```

$$\frac{a^{16}\,x^{3}}{3}+\frac{8}{3}\,a^{15}\,b\,x^{6}+\frac{40}{3}\,a^{14}\,b^{2}\,x^{9}+\frac{140}{3}\,a^{13}\,b^{3}\,x^{12}+\frac{364}{3}\,a^{12}\,b^{4}\,x^{15}+\frac{728}{3}\,a^{11}\,b^{5}\,x^{18}+\frac{1144}{3}\,a^{10}\,b^{6}\,x^{21}+\frac{1430}{3}\,a^{9}\,b^{7}\,x^{24}+\frac{1430}{3}\,a^{8}\,b^{8}\,x^{27}+\frac{1144}{3}\,a^{7}\,b^{9}\,x^{30}+\frac{728}{3}\,a^{6}\,b^{10}\,x^{33}+\frac{364}{3}\,a^{5}\,b^{11}\,x^{36}+\frac{140}{3}\,a^{4}\,b^{12}\,x^{39}+\frac{40}{3}\,a^{3}\,b^{13}\,x^{42}+\frac{8}{3}\,a^{2}\,b^{14}\,x^{45}+\frac{1}{3}\,a\,b^{15}\,x^{48}+\frac{b^{16}\,x^{51}}{51}$$

Rational Function Integration Problem 2

$$\int x^p \left(a\, x^n + b\, x^{m\, n+n+p+1}\right)^m \, dx$$

- *Rubi* knows and takes advantage of the general rule for arbitrary m, n and p:

```
Int[x^p (a x^n + b x^(m n+n+p+1))^m, x]
```

$$\frac{x^{-(1+m)\, n} \left(x^n \left(a + b\, x^{1+m\, n+p}\right)\right)^{1+m}}{b\, (1+m)\, (1+m\, n+p)}$$

```
Int[x^p (a x^n + b x^(12 n+n+p+1))^12, x]
```

$$\frac{\left(a + b\, x^{1+12\, n+p}\right)^{13}}{13\, b\, (1+12\, n+p)}$$

```
Int[x^24 (a x + b x^38)^12, x]
```

$$\frac{\left(a + b\, x^{37}\right)^{13}}{481\, b}$$

- *Mathematica* knows the rule for arbitrary m, n and p, but needlessly expands the integrand when m is a positive integer:

$$\int x^p \left(a\, x^n + b\, x^{m\, n+n+p+1}\right)^m \, dx$$

$$\frac{x^{-(1+m)\, n} \left(x^n \left(a + b\, x^{1+m\, n+p}\right)\right)^{1+m}}{b\, (1+m)\, (1+m\, n+p)}$$

$$\int x^p \left(a\, x^n + b\, x^{12\, n+n+p+1}\right)^{12} \, dx$$

$$\frac{1}{13\, (1+12\, n+p)} x^{1+12\, n+p} \left(13\, a^{12} + 78\, a^{11}\, b\, x^{1+12\, n+p} + 286\, a^3\, b^9\, x^{9\, (1+12\, n+p)} + 78\, a^2\, b^{10}\, x^{10\, (1+12\, n+p)} + 13\, a\, b^{11}\, x^{11\, (1+12\, n+p)} + b^{12}\, x^{12\, (1+12\, n+p)} + 286\, a^{10}\, b^2\, x^{2+24\, n+2\, p} + 715\, a^9\, b^3\, x^{3+36\, n+3\, p} + 1287\, a^8\, b^4\, x^{4+48\, n+4\, p} + 1716\, a^7\, b^5\, x^{5+60\, n+5\, p} + 1716\, a^6\, b^6\, x^{6+72\, n+6\, p} + 1287\, a^5\, b^7\, x^{7+84\, n+7\, p} + 715\, a^4\, b^8\, x^{8+96\, n+8\, p}\right)$$

$$\int x^{24} \left(a\, x + b\, x^{38}\right)^{12} \, dx$$

$$\frac{a^{12}\, x^{37}}{37} + \frac{6}{37} a^{11}\, b\, x^{74} + \frac{22}{37} a^{10}\, b^2\, x^{111} + \frac{55}{37} a^9\, b^3\, x^{148} + \frac{99}{37} a^8\, b^4\, x^{185} + \frac{132}{37} a^7\, b^5\, x^{222} + \frac{132}{37} a^6\, b^6\, x^{259} + \frac{99}{37} a^5\, b^7\, x^{296} + \frac{55}{37} a^4\, b^8\, x^{333} + \frac{22}{37} a^3\, b^9\, x^{370} + \frac{6}{37} a^2\, b^{10}\, x^{407} + \frac{1}{37} a\, b^{11}\, x^{444} + \frac{b^{12}\, x^{481}}{481}$$

- *Maple* does *not* know the rule for arbitrary m, n and p, and needlessly expands the integrand when m is a positive integer:

```
int(x^p*(a*x^n+b*x^(m*n+n+p+1))^m, x);
```

$$\int x^p \left(a\,x^n + b\,x^{m\,n+n+p+1}\right)^m dx$$

```
int(x^p*(a*x^n+b*x^(12*n+n+p+1))^12, x);
```

$$\frac{1}{13\,(1+12\,n+p)}\left(13\,a^{12}\,x^{1+12\,n+p} + 78\,a^{11}\,b\,x^{2+24\,n+2\,p} + 286\,a^{10}\,b^2\,x^{3+36\,n+3\,p} + 715\,a^9\,b^3\,x^{4+48\,n+4\,p} + 1287\,a^8\,b^4\,x^{5+60\,n+5\,p} + 1716\,a^7\,b^5\,x^{6+72\,n+6\,p} + 1716\,a^6\,b^6\,x^{7+84\,n+7\,p} + 1287\,a^5\,b^7\,x^{8+96\,n+8\,p} + 715\,a^4\,b^8\,x^{9+108\,n+9\,p} + 286\,a^3\,b^9\,x^{10+120\,n+10\,p} + 78\,a^2\,b^{10}\,x^{11+132\,n+11\,p} + 13\,a\,b^{11}\,x^{12+144\,n+12\,p} + b^{12}\,x^{13+156\,n+13\,p}\right)$$

```
int(x^24*(a*x+b*x^38)^12, x);
```

$$\frac{a^{12}\,x^{37}}{37} + \frac{6}{37}\,a^{11}\,b\,x^{74} + \frac{22}{37}\,a^{10}\,b^2\,x^{111} + \frac{55}{37}\,a^9\,b^3\,x^{148} + \frac{99}{37}\,a^8\,b^4\,x^{185} + \frac{132}{37}\,a^7\,b^5\,x^{222} + \frac{132}{37}\,a^6\,b^6\,x^{259} + \frac{99}{37}\,a^5\,b^7\,x^{296} + \frac{55}{37}\,a^4\,b^8\,x^{333} + \frac{22}{37}\,a^3\,b^9\,x^{370} + \frac{6}{37}\,a^2\,b^{10}\,x^{407} + \frac{1}{37}\,a\,b^{11}\,x^{444} + \frac{b^{12}\,x^{481}}{481}$$

Rational Function Integration Problem 3

$$\int (b + 2\,c\,x)\,\left(a + b\,x + c\,x^2\right)^m\,dx$$

- *Rubi* knows and takes advantage of the general rule for arbitrary m:

```
Int[(b + 2 c x) (a + b x + c x^2)^m, x]
```

$$\frac{\left(a + b\,x + c\,x^2\right)^{1+m}}{1 + m}$$

```
Int[(b + 2 c x) (a + b x + c x^2)^12, x]
```

$$\frac{1}{13}\left(a + b\,x + c\,x^2\right)^{13}$$

- *Mathematica* knows the rule for arbitrary m, but needlessly expands the integrand when m is a positive integer:

$$\int (b + 2\,c\,x)\,\left(a + b\,x + c\,x^2\right)^m\,dx$$

$$\frac{(a + x\,(b + c\,x))^{1+m}}{1 + m}$$

$$\int (b + 2\,c\,x)\,\left(a + b\,x + c\,x^2\right)^{12}\,dx$$

$$a^{12}\,x\,(b + c\,x) + 6\,a^{11}\,x^2\,(b + c\,x)^2 + 22\,a^{10}\,x^3\,(b + c\,x)^3 + 55\,a^9\,x^4\,(b + c\,x)^4 + 99\,a^8\,x^5\,(b + c\,x)^5 + 132\,a^7\,x^6\,(b + c\,x)^6 + 132\,a^6\,x^7\,(b + c\,x)^7 + 99\,a^5\,x^8\,(b + c\,x)^8 + 55\,a^4\,x^9\,(b + c\,x)^9 + 22\,a^3\,x^{10}\,(b + c\,x)^{10} + 6\,a^2\,x^{11}\,(b + c\,x)^{11} + a\,x^{12}\,(b + c\,x)^{12} + \frac{1}{13}\,x^{13}\,(b + c\,x)^{13}$$

- *Maple* knows the rule for arbitrary m, but needlessly expands the integrand when m is a positive integer:

```
int((b+2*c*x)*(a+b*x+c*x^2)^m, x);
```

$$\frac{\left(a + b\,x + c\,x^2\right)^{m+1}}{m + 1}$$

```
int((b+2*c*x)*(a+b*x+c*x^2)^12, x);
```

$$a^{12} b x + 6 a^{11} b^2 x^2 + a^{12} c x^2 + 22 a^{10} b^3 x^3 + 12 a^{11} b c x^3 + 55 a^9 b^4 x^4 + 66 a^{10} b^2 c x^4 + 6 a^{11} c^2 x^4 + 99 a^8 b^5 x^5 +$$

$$220 a^9 b^3 c x^5 + 66 a^{10} b c^2 x^5 + 132 a^7 b^6 x^6 + 495 a^8 b^4 c x^6 + 330 a^9 b^2 c^2 x^6 + 22 a^{10} c^3 x^6 + 132 a^6 b^7 x^7 +$$

$$792 a^7 b^5 c x^7 + 990 a^8 b^3 c^2 x^7 + 220 a^9 b c^3 x^7 + 99 a^5 b^8 x^8 + 924 a^6 b^6 c x^8 + 1980 a^7 b^4 c^2 x^8 + 990 a^8 b^2 c^3 x^8 +$$

$$55 a^9 c^4 x^8 + 55 a^4 b^9 x^9 + 792 a^5 b^7 c x^9 + 2772 a^6 b^5 c^2 x^9 + 2640 a^7 b^3 c^3 x^9 + 495 a^8 b c^4 x^9 + 22 a^3 b^{10} x^{10} +$$

$$495 a^4 b^8 c x^{10} + 2772 a^5 b^6 c^2 x^{10} + 4620 a^6 b^4 c^3 x^{10} + 1980 a^7 b^2 c^4 x^{10} + 99 a^8 c^5 x^{10} + 6 a^2 b^{11} x^{11} + 220 a^3 b^9 c x^{11} +$$

$$1980 a^4 b^7 c^2 x^{11} + 5544 a^5 b^5 c^3 x^{11} + 4620 a^6 b^3 c^4 x^{11} + 792 a^7 b c^5 x^{11} + a b^{12} x^{12} + 66 a^2 b^{10} c x^{12} + 990 a^3 b^8 c^2 x^{12} +$$

$$4620 a^4 b^6 c^3 x^{12} + 6930 a^5 b^4 c^4 x^{12} + 2772 a^6 b^2 c^5 x^{12} + 132 a^7 c^6 x^{12} + \frac{b^{13} x^{13}}{13} + 12 a b^{11} c x^{13} + 330 a^2 b^9 c^2 x^{13} +$$

$$2640 a^3 b^7 c^3 x^{13} + 6930 a^4 b^5 c^4 x^{13} + 5544 a^5 b^3 c^5 x^{13} + 924 a^6 b c^6 x^{13} + b^{12} c x^{14} + 66 a b^{10} c^2 x^{14} +$$

$$990 a^2 b^8 c^3 x^{14} + 4620 a^3 b^6 c^4 x^{14} + 6930 a^4 b^4 c^5 x^{14} + 2772 a^5 b^2 c^6 x^{14} + 132 a^6 c^7 x^{14} + 6 b^{11} c^2 x^{15} +$$

$$220 a b^9 c^3 x^{15} + 1980 a^2 b^7 c^4 x^{15} + 5544 a^3 b^5 c^5 x^{15} + 4620 a^4 b^3 c^6 x^{15} + 792 a^5 b c^7 x^{15} + 22 b^{10} c^3 x^{16} +$$

$$495 a b^8 c^4 x^{16} + 2772 a^2 b^6 c^5 x^{16} + 4620 a^3 b^4 c^6 x^{16} + 1980 a^4 b^2 c^7 x^{16} + 99 a^5 c^8 x^{16} + 55 b^9 c^4 x^{17} + 792 a b^7 c^5 x^{17} +$$

$$2772 a^2 b^5 c^6 x^{17} + 2640 a^3 b^3 c^7 x^{17} + 495 a^4 b c^8 x^{17} + 99 b^8 c^5 x^{18} + 924 a b^6 c^6 x^{18} + 1980 a^2 b^4 c^7 x^{18} +$$

$$990 a^3 b^2 c^8 x^{18} + 55 a^4 c^9 x^{18} + 132 b^7 c^6 x^{19} + 792 a b^5 c^7 x^{19} + 990 a^2 b^3 c^8 x^{19} + 220 a^3 b c^9 x^{19} + 132 b^6 c^7 x^{20} +$$

$$495 a b^4 c^8 x^{20} + 330 a^2 b^2 c^9 x^{20} + 22 a^3 c^{10} x^{20} + 99 b^5 c^8 x^{21} + 220 a b^3 c^9 x^{21} + 66 a^2 b c^{10} x^{21} + 55 b^4 c^9 x^{22} +$$

$$66 a b^2 c^{10} x^{22} + 6 a^2 c^{11} x^{22} + 22 b^3 c^{10} x^{23} + 12 a b c^{11} x^{23} + 6 b^2 c^{11} x^{24} + a c^{12} x^{24} + b c^{12} x^{25} + \frac{c^{13} x^{26}}{13}$$

Rational Function Integration Problem 4

$$\int (a + b x)^{n-1} \left(c + d (a + b x)^n\right)^m \, dx$$

- *Rubi* knows and takes advantage of the general rule for arbitrary m and n:

```
Int[(a + b x)^(n-1) (c + d (a + b x)^n)^m, x]
```

$$\frac{\left(c + d (a + b x)^n\right)^{1+m}}{b d (1 + m) n}$$

```
Int[(a + b x)^(n-1) (c + d (a + b x)^n)^8, x]
```

$$\frac{\left(c + d (a + b x)^n\right)^9}{9 b d n}$$

```
Int[(a + b x)^2 (c + d (a + b x)^3)^8, x]
```

$$\frac{\left(c + d (a + b x)^3\right)^9}{27 b d}$$

- *Mathematica* knows the rule for arbitrary m and n, but needlessly expands the integrand when m is a positive integer:

$$\int (a + b x)^{n-1} \left(c + d (a + b x)^n\right)^m \, dx$$

$$\frac{\left(c + d (a + b x)^n\right)^{1+m}}{b d n + b d m n}$$

$$\int (a + b x)^{n-1} \left(c + d (a + b x)^n\right)^8 \, dx$$

$$\frac{1}{9 b n} (a + b x)^n \left(9 c^8 + 36 c^7 d (a + b x)^n + 84 c^6 d^2 (a + b x)^{2 n} + 126 c^5 d^3 (a + b x)^{3 n} + 126 c^4 d^4 (a + b x)^{4 n} + 84 c^3 d^5 (a + b x)^{5 n} + 36 c^2 d^6 (a + b x)^{6 n} + 9 c d^7 (a + b x)^{7 n} + d^8 (a + b x)^{8 n}\right)$$

$$\int (a + b x)^2 \left(c + d (a + b x)^3\right)^8 \, dx$$

$$a^2 \left(c+a^3 d\right)^8 x+a b \left(c+a^3 d\right)^7 \left(c+13 a^3 d\right) x^2+$$
$$\frac{1}{3} b^2 \left(c+a^3 d\right)^6 \left(c^2+74 a^3 c d+325 a^6 d^2\right) x^3+2 a^2 b^3 d \left(c+a^3 d\right)^5 \left(10 c^2+146 a^3 c d+325 a^6 d^2\right) x^4+$$
$$2 a b^4 d \left(c+a^3 d\right)^4 \left(4 c^3+180 a^3 c^2 d+1104 a^6 c d^2+1495 a^9 d^3\right) x^5+$$
$$\frac{2}{3} b^5 d \left(c+a^3 d\right)^3 \left(2 c^4+386 a^3 c^3 d+5304 a^6 c^2 d^2+17\,963 a^9 c d^3+16\,445 a^{12} d^4\right) x^6+$$
$$2 a^2 b^6 d^2 \left(c+a^3 d\right)^2 \left(56 c^4+1736 a^3 c^3 d+11\,487 a^6 c^2 d^2+24\,794 a^9 c d^3+16\,445 a^{12} d^4\right) x^7+$$
$$a b^7 d^2 \left(28 c^6+2310 a^3 c^5 d+30\,030 a^6 c^4 d^2+136\,136 a^9 c^3 d^3+271\,320 a^{12} c^2 d^4+245\,157 a^{15} c d^5+82\,225 a^{18} d^6\right) x^8+$$
$$\frac{1}{9} b^8 d^2$$
$$\left(28 c^6+9240 a^3 c^5 d+210\,210 a^6 c^4 d^2+1\,361\,360 a^9 c^3 d^3+3\,527\,160 a^{12} c^2 d^4+3\,922\,512 a^{15} c d^5+1\,562\,275 a^{18} d^6\right)$$
$$x^9+a^2 b^9 d^3 \left(308 c^5+14\,014 a^3 c^4 d+136\,136 a^6 c^3 d^2+470\,288 a^9 c^2 d^3+653\,752 a^{12} c d^4+312\,455 a^{15} d^5\right) x^{10}+$$
$$a b^{10} d^3 \left(56 c^5+6370 a^3 c^4 d+99\,008 a^6 c^3 d^2+470\,288 a^9 c^2 d^3+832\,048 a^{12} c d^4+482\,885 a^{15} d^5\right) x^{11}+$$
$$\frac{2}{3} b^{11} d^3 \left(7 c^5+3185 a^3 c^4 d+86\,632 a^6 c^3 d^2+587\,860 a^9 c^2 d^3+1\,352\,078 a^{12} c d^4+965\,770 a^{15} d^5\right) x^{12}+$$
$$2 a^2 b^{12} d^4 \left(245 c^4+13\,328 a^3 c^3 d+135\,660 a^6 c^2 d^2+416\,024 a^9 c d^3+371\,450 a^{12} d^4\right) x^{13}+$$
$$2 a b^{13} d^4 \left(35 c^4+4760 a^3 c^3 d+77\,520 a^6 c^2 d^2+326\,876 a^9 c d^3+371\,450 a^{12} d^4\right) x^{14}+$$
$$\frac{2}{3} b^{14} d^4 \left(7 c^4+3808 a^3 c^3 d+108\,528 a^6 c^2 d^2+653\,752 a^9 c d^3+965\,770 a^{12} d^4\right) x^{15}+$$
$$17 a^2 b^{15} d^5 \left(28 c^3+1596 a^3 c^2 d+14\,421 a^6 c d^2+28\,405 a^9 d^3\right) x^{16}+$$
$$a b^{16} d^5 \left(56 c^3+7980 a^3 c^2 d+115\,368 a^6 c d^2+312\,455 a^9 d^3\right) x^{17}+$$
$$\frac{1}{9} b^{17} d^5 \left(28 c^3+15\,960 a^3 c^2 d+403\,788 a^6 c d^2+1\,562\,275 a^9 d^3\right) x^{18}+$$
$$a^2 b^{18} d^6 \left(280 c^2+14\,168 a^3 c d+82\,225 a^6 d^2\right) x^{19}+2 a b^{19} d^6 \left(14 c^2+1771 a^3 c d+16\,445 a^6 d^2\right) x^{20}+$$
$$\frac{2}{3} b^{20} d^6 \left(2 c^2+1012 a^3 c d+16\,445 a^6 d^2\right) x^{21}+46 a^2 b^{21} d^7 \left(2 c+65 a^3 d\right) x^{22}+$$
$$2 a b^{22} d^7 \left(4 c+325 a^3 d\right) x^{23}+\frac{1}{3} b^{23} d^7 \left(c+325 a^3 d\right) x^{24}+13 a^2 b^{24} d^8 x^{25}+a b^{25} d^8 x^{26}+\frac{1}{27} b^{26} d^8 x^{27}$$

- *Maple* knows the rule for arbitrary m and n, but needlessly expands the integrand when m is a positive integer:

```
int((a+b*x)^(n-1)*(c+d*(a+b*x)^n)^m, x);
```

$$\frac{\left(c+d\,(a+b\,x)^n\right)\,\left(c+d\,(a+b\,x)^n\right)^m}{b\,n\,d\,(1+m)}$$

```
int((a+b*x)^(n-1)*(c+d*(a+b*x)^n)^8, x);
```

$$\frac{c^8\,(a+b\,x)^n}{b\,n}+\frac{4\,c^7\,d\,(a+b\,x)^{2\,n}}{b\,n}+\frac{28\,c^6\,d^2\,(a+b\,x)^{3\,n}}{3\,b\,n}+\frac{14\,c^5\,d^3\,(a+b\,x)^{4\,n}}{b\,n}+$$
$$\frac{14\,c^4\,d^4\,(a+b\,x)^{5\,n}}{b\,n}+\frac{28\,c^3\,d^5\,(a+b\,x)^{6\,n}}{3\,b\,n}+\frac{4\,c^2\,d^6\,(a+b\,x)^{7\,n}}{b\,n}+\frac{c\,d^7\,(a+b\,x)^{8\,n}}{b\,n}+\frac{d^8\,(a+b\,x)^{9\,n}}{9\,b\,n}$$

```
int((a+b*x)^2*(c+d*(a+b*x)^3)^8, x);
```

$$a^{2} c^{8} x + 8 a^{5} c^{7} d x + 28 a^{8} c^{6} d^{2} x + 56 a^{11} c^{5} d^{3} x + 70 a^{14} c^{4} d^{4} x + 56 a^{17} c^{3} d^{5} x + 28 a^{20} c^{2} d^{6} x + 8 a^{23} c d^{7} x +$$

$$a^{26} d^{8} x + a b c^{8} x^{2} + 20 a^{4} b c^{7} d x^{2} + 112 a^{7} b c^{6} d^{2} x^{2} + 308 a^{10} b c^{5} d^{3} x^{2} + 490 a^{13} b c^{4} d^{4} x^{2} + 476 a^{16} b c^{3} d^{5} x^{2} +$$

$$280 a^{19} b c^{2} d^{6} x^{2} + 92 a^{22} b c d^{7} x^{2} + 13 a^{25} b d^{8} x^{2} + \frac{1}{3} b^{2} c^{8} x^{3} + \frac{80}{3} a^{3} b^{2} c^{7} d x^{3} + \frac{784}{3} a^{6} b^{2} c^{6} d^{2} x^{3} +$$

$$\frac{3080}{3} a^{9} b^{2} c^{5} d^{3} x^{3} + \frac{6370}{3} a^{12} b^{2} c^{4} d^{4} x^{3} + \frac{7616}{3} a^{15} b^{2} c^{3} d^{5} x^{3} + \frac{5320}{3} a^{18} b^{2} c^{2} d^{6} x^{3} + \frac{2024}{3} a^{21} b^{2} c d^{7} x^{3} +$$

$$\frac{325}{3} a^{24} b^{2} d^{8} x^{3} + 20 a^{2} b^{3} c^{7} d x^{4} + 392 a^{5} b^{3} c^{6} d^{2} x^{4} + 2310 a^{8} b^{3} c^{5} d^{3} x^{4} + 6370 a^{11} b^{3} c^{4} d^{4} x^{4} + 9520 a^{14} b^{3} c^{3} d^{5} x^{4} +$$

$$7980 a^{17} b^{3} c^{2} d^{6} x^{4} + 3542 a^{20} b^{3} c d^{7} x^{4} + 650 a^{23} b^{3} d^{8} x^{4} + 8 a b^{4} c^{7} d x^{5} + 392 a^{4} b^{4} c^{6} d^{2} x^{5} + 3696 a^{7} b^{4} c^{5} d^{3} x^{5} +$$

$$14\,014 a^{10} b^{4} c^{4} d^{4} x^{5} + 26\,656 a^{13} b^{4} c^{3} d^{5} x^{5} + 27\,132 a^{16} b^{4} c^{2} d^{6} x^{5} + 14\,168 a^{19} b^{4} c d^{7} x^{5} + 2990 a^{22} b^{4} d^{8} x^{5} +$$

$$\frac{4}{3} b^{5} c^{7} d x^{6} + \frac{784}{3} a^{3} b^{5} c^{6} d^{2} x^{6} + 4312 a^{6} b^{5} c^{5} d^{3} x^{6} + \frac{70\,070}{3} a^{9} b^{5} c^{4} d^{4} x^{6} + \frac{173\,264}{3} a^{12} b^{5} c^{3} d^{5} x^{6} +$$

$$72\,352 a^{15} b^{5} c^{2} d^{6} x^{6} + \frac{134\,596}{3} a^{18} b^{5} c d^{7} x^{6} + \frac{32\,890}{3} a^{21} b^{5} d^{8} x^{6} + 112 a^{2} b^{6} c^{6} d^{2} x^{7} + 3696 a^{5} b^{6} c^{5} d^{3} x^{7} +$$

$$30\,030 a^{8} b^{6} c^{4} d^{4} x^{7} + 99\,008 a^{11} b^{6} c^{3} d^{5} x^{7} + 155\,040 a^{14} b^{6} c^{2} d^{6} x^{7} + 115\,368 a^{17} b^{6} c d^{7} x^{7} + 32\,890 a^{20} b^{6} d^{8} x^{7} +$$

$$28 a b^{7} c^{6} d^{2} x^{8} + 2310 a^{4} b^{7} c^{5} d^{3} x^{8} + 30\,030 a^{7} b^{7} c^{4} d^{4} x^{8} + 136\,136 a^{10} b^{7} c^{3} d^{5} x^{8} + 271\,320 a^{13} b^{7} c^{2} d^{6} x^{8} +$$

$$245\,157 a^{16} b^{7} c d^{7} x^{8} + 82\,225 a^{19} b^{7} d^{8} x^{8} + \frac{28}{9} b^{8} c^{6} d^{2} x^{9} + \frac{3080}{3} a^{3} b^{8} c^{5} d^{3} x^{9} + \frac{70\,070}{3} a^{6} b^{8} c^{4} d^{4} x^{9} +$$

$$\frac{1\,361\,360}{9} a^{9} b^{8} c^{3} d^{5} x^{9} + \frac{1\,175\,720}{3} a^{12} b^{8} c^{2} d^{6} x^{9} + \frac{1\,307\,504}{3} a^{15} b^{8} c d^{7} x^{9} + \frac{1\,562\,275}{9} a^{18} b^{8} d^{8} x^{9} + 308 a^{2} b^{9} c^{5} d^{3} x^{10} +$$

$$14\,014 a^{5} b^{9} c^{4} d^{4} x^{10} + 136\,136 a^{8} b^{9} c^{3} d^{5} x^{10} + 470\,288 a^{11} b^{9} c^{2} d^{6} x^{10} + 653\,752 a^{14} b^{9} c d^{7} x^{10} + 312\,455 a^{17} b^{9} d^{8} x^{10} +$$

$$56 a b^{10} c^{5} d^{3} x^{11} + 6370 a^{4} b^{10} c^{4} d^{4} x^{11} + 99\,008 a^{7} b^{10} c^{3} d^{5} x^{11} + 470\,288 a^{10} b^{10} c^{2} d^{6} x^{11} + 832\,048 a^{13} b^{10} c d^{7} x^{11} +$$

$$482\,885 a^{16} b^{10} d^{8} x^{11} + \frac{14}{3} b^{11} c^{5} d^{3} x^{12} + \frac{6370}{3} a^{3} b^{11} c^{4} d^{4} x^{12} + \frac{173\,264}{3} a^{6} b^{11} c^{3} d^{5} x^{12} + \frac{1\,175\,720}{3} a^{9} b^{11} c^{2} d^{6} x^{12} +$$

$$\frac{2\,704\,156}{3} a^{12} b^{11} c d^{7} x^{12} + \frac{1\,931\,540}{3} a^{15} b^{11} d^{8} x^{12} + 490 a^{2} b^{12} c^{4} d^{4} x^{13} + 26\,656 a^{5} b^{12} c^{3} d^{5} x^{13} +$$

$$271\,320 a^{8} b^{12} c^{2} d^{6} x^{13} + 832\,048 a^{11} b^{12} c d^{7} x^{13} + 742\,900 a^{14} b^{12} d^{8} x^{13} + 70 a b^{13} c^{4} d^{4} x^{14} + 9520 a^{4} b^{13} c^{3} d^{5} x^{14} +$$

$$155\,040 a^{7} b^{13} c^{2} d^{6} x^{14} + 653\,752 a^{10} b^{13} c d^{7} x^{14} + 742\,900 a^{13} b^{13} d^{8} x^{14} + \frac{14}{3} b^{14} c^{4} d^{4} x^{15} + \frac{7616}{3} a^{3} b^{14} c^{3} d^{5} x^{15} +$$

$$72\,352 a^{6} b^{14} c^{2} d^{6} x^{15} + \frac{1\,307\,504}{3} a^{9} b^{14} c d^{7} x^{15} + \frac{1\,931\,540}{3} a^{12} b^{14} d^{8} x^{15} + 476 a^{2} b^{15} c^{3} d^{5} x^{16} + 27\,132 a^{5} b^{15} c^{2} d^{6} x^{16} +$$

$$245\,157 a^{8} b^{15} c d^{7} x^{16} + 482\,885 a^{11} b^{15} d^{8} x^{16} + 56 a b^{16} c^{3} d^{5} x^{17} + 7980 a^{4} b^{16} c^{2} d^{6} x^{17} + 115\,368 a^{7} b^{16} c d^{7} x^{17} +$$

$$312\,455 a^{10} b^{16} d^{8} x^{17} + \frac{28}{9} b^{17} c^{3} d^{5} x^{18} + \frac{5320}{3} a^{3} b^{17} c^{2} d^{6} x^{18} + \frac{134\,596}{3} a^{6} b^{17} c d^{7} x^{18} + \frac{1\,562\,275}{9} a^{9} b^{17} d^{8} x^{18} +$$

$$280 a^{2} b^{18} c^{2} d^{6} x^{19} + 14\,168 a^{5} b^{18} c d^{7} x^{19} + 82\,225 a^{8} b^{18} d^{8} x^{19} + 28 a b^{19} c^{2} d^{6} x^{20} + 3542 a^{4} b^{19} c d^{7} x^{20} +$$

$$32\,890 a^{7} b^{19} d^{8} x^{20} + \frac{4}{3} b^{20} c^{2} d^{6} x^{21} + \frac{2024}{3} a^{3} b^{20} c d^{7} x^{21} + \frac{32\,890}{3} a^{6} b^{20} d^{8} x^{21} + 92 a^{2} b^{21} c d^{7} x^{22} + 2990 a^{5} b^{21} d^{8} x^{22} +$$

$$8 a b^{22} c d^{7} x^{23} + 650 a^{4} b^{22} d^{8} x^{23} + \frac{1}{3} b^{23} c d^{7} x^{24} + \frac{325}{3} a^{3} b^{23} d^{8} x^{24} + 13 a^{2} b^{24} d^{8} x^{25} + a b^{25} d^{8} x^{26} + \frac{1}{27} b^{26} d^{8} x^{27}$$

Rational Function Integration Problem 5

$$\int \frac{x^{18}}{(a + b x)^{20}} \, dx$$

- *Rubi* knows and takes advantage of the general rule for arbitrary n:

$$\text{Int}\left[\frac{x^{n-2}}{(a+b x)^n}, x\right]$$

$$\frac{x^{-1+n} (a+b x)^{1-n}}{a (-1+n)}$$

$$\text{Int}\left[\frac{x^{18}}{(a+b x)^{20}}, x\right]$$

$$\frac{x^{19}}{19 a (a+b x)^{19}}$$

- *Mathematica* knows the rule for arbitrary n, but needlessly expands the integrand when n is an integer:

$$\int \frac{x^{n-2}}{(a+b x)^n} \, dx$$

$$\frac{x^{-1+n} (a+b x)^{1-n}}{a (-1+n)}$$

$$\int \frac{x^{18}}{(a+b x)^{20}} \, dx$$

$$-\frac{1}{19 b^{19} (a+b x)^{19}} \left(a^{18} + 19 a^{17} b x + 171 a^{16} b^2 x^2 + 969 a^{15} b^3 x^3 + 3876 a^{14} b^4 x^4 + 11\,628 a^{13} b^5 x^5 + 27\,132 a^{12} b^6 x^6 + 50\,388 a^{11} b^7 x^7 + 75\,582 a^{10} b^8 x^8 + 92\,378 a^9 b^9 x^9 + 92\,378 a^8 b^{10} x^{10} + 75\,582 a^7 b^{11} x^{11} + 50\,388 a^6 b^{12} x^{12} + 27\,132 a^5 b^{13} x^{13} + 11\,628 a^4 b^{14} x^{14} + 3876 a^3 b^{15} x^{15} + 969 a^2 b^{16} x^{16} + 171 a b^{17} x^{17} + 19 b^{18} x^{18}\right)$$

- *Maple* knows the rule for arbitrary n, but needlessly expands the integrand when n is an integer:

```
int(x^(n-2)/(a+b*x)^n, x);
```

$$\frac{x^{-1+n} (a+b x)^{1-n}}{a (-1+n)}$$

```
int(x^18/(a+b*x)^20, x);
```

$$-\frac{a^{18}}{19 b^{19} (a+b x)^{19}}+\frac{a^{17}}{b^{19} (a+b x)^{18}}-\frac{9 a^{16}}{b^{19} (a+b x)^{17}}+\frac{51 a^{15}}{b^{19} (a+b x)^{16}}-\frac{204 a^{14}}{b^{19} (a+b x)^{15}}+\frac{612 a^{13}}{b^{19} (a+b x)^{14}}-$$

$$\frac{1428 a^{12}}{b^{19} (a+b x)^{13}}+\frac{2652 a^{11}}{b^{19} (a+b x)^{12}}-\frac{3978 a^{10}}{b^{19} (a+b x)^{11}}+\frac{4862 a^{9}}{b^{19} (a+b x)^{10}}-\frac{4862 a^{8}}{b^{19} (a+b x)^{9}}+\frac{3978 a^{7}}{b^{19} (a+b x)^{8}}-$$

$$\frac{2652 a^{6}}{b^{19} (a+b x)^{7}}+\frac{1428 a^{5}}{b^{19} (a+b x)^{6}}-\frac{612 a^{4}}{b^{19} (a+b x)^{5}}+\frac{204 a^{3}}{b^{19} (a+b x)^{4}}-\frac{51 a^{2}}{b^{19} (a+b x)^{3}}+\frac{9 a}{b^{19} (a+b x)^{2}}-\frac{1}{b^{19} (a+b x)}$$

Rational Function Integration Problem 6

$$\int \frac{x^{17}}{(a+b\,x)^{20}}\,dx$$

- *Rubi* knows and takes advantage of the general rule for arbitrary n:

```
Int[x^(n-3)/(a+b x)^n, x]
```

$$-\frac{x^{-2+n}\,(a+b\,x)^{1-n}}{a\,(1-n)}+\frac{x^{-2+n}\,(a+b\,x)^{2-n}}{a^2\,(1-n)\,(2-n)}$$

```
Int[x^17/(a+b x)^20, x]
```

$$\frac{x^{18}}{19\,a\,(a+b\,x)^{19}}+\frac{x^{18}}{342\,a^2\,(a+b\,x)^{18}}$$

- *Mathematica* knows the rule for arbitrary n, but needlessly expands the integrand when n is an integer:

$$\int \frac{x^{n-3}}{(a+b\,x)^n}\,dx$$

$$\frac{x^{-2+n}\,(a+b\,x)^{1-n}\,(a\,(-1+n)+b\,x)}{a^2\,(-2+n)\,(-1+n)}$$

$$\int \frac{x^{17}}{(a+b\,x)^{20}}\,dx$$

$$-\frac{1}{342\,b^{18}\,(a+b\,x)^{19}}\left(a^{17}+19\,a^{16}\,b\,x+171\,a^{15}\,b^2\,x^2+969\,a^{14}\,b^3\,x^3+3876\,a^{13}\,b^4\,x^4+11\,628\,a^{12}\,b^5\,x^5+\right.$$
$$27\,132\,a^{11}\,b^6\,x^6+50\,388\,a^{10}\,b^7\,x^7+75\,582\,a^9\,b^8\,x^8+92\,378\,a^8\,b^9\,x^9+92\,378\,a^7\,b^{10}\,x^{10}+75\,582\,a^6\,b^{11}\,x^{11}+$$
$$\left.50\,388\,a^5\,b^{12}\,x^{12}+27\,132\,a^4\,b^{13}\,x^{13}+11\,628\,a^3\,b^{14}\,x^{14}+3876\,a^2\,b^{15}\,x^{15}+969\,a\,b^{16}\,x^{16}+171\,b^{17}\,x^{17}\right)$$

- *Maple* knows the rule for arbitrary n, but needlessly expands the integrand when n is an integer:

```
int(x^(n-2)/(a+b*x)^n, x);
```

$$\frac{x^{-2+n}\,(a+b\,x)^{1-n}\,(a\,(-1+n)+b\,x)}{a^2\,(-2+n)\,(-1+n)}$$

```
int(x^17/(a+b*x)^20, x);
```

$$\frac{a^{17}}{19\,b^{18}\,(a+b\,x)^{19}}-\frac{17\,a^{16}}{18\,b^{18}\,(a+b\,x)^{18}}+\frac{8\,a^{15}}{b^{18}\,(a+b\,x)^{17}}-\frac{85\,a^{14}}{2\,b^{18}\,(a+b\,x)^{16}}+\frac{476\,a^{13}}{3\,b^{18}\,(a+b\,x)^{15}}-\frac{442\,a^{12}}{b^{18}\,(a+b\,x)^{14}}+$$

$$\frac{952\,a^{11}}{b^{18}\,(a+b\,x)^{13}}-\frac{4862\,a^{10}}{3\,b^{18}\,(a+b\,x)^{12}}+\frac{2210\,a^{9}}{b^{18}\,(a+b\,x)^{11}}-\frac{2431\,a^{8}}{b^{18}\,(a+b\,x)^{10}}+\frac{19\,448\,a^{7}}{9\,b^{18}\,(a+b\,x)^{9}}-\frac{1547\,a^{6}}{b^{18}\,(a+b\,x)^{8}}+$$

$$\frac{884\,a^{5}}{b^{18}\,(a+b\,x)^{7}}-\frac{1190\,a^{4}}{3\,b^{18}\,(a+b\,x)^{6}}+\frac{136\,a^{3}}{b^{18}\,(a+b\,x)^{5}}-\frac{34\,a^{2}}{b^{18}\,(a+b\,x)^{4}}+\frac{17\,a}{3\,b^{18}\,(a+b\,x)^{3}}-\frac{1}{2\,b^{18}\,(a+b\,x)^{2}}$$

Rational Function Integration Problem 7

$$\int \frac{1}{a + b x^2} \, dx$$

- *Rubi* knows and always uses the symmetric rules:

$$\left\{\text{Int}\left[\frac{1}{a + b x^2}, x\right], \text{Int}\left[\frac{1}{a - b x^2}, x\right]\right\}$$

$$\left\{\frac{\text{ArcTan}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{\sqrt{a}\,\sqrt{b}}, \frac{\text{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{\sqrt{a}\,\sqrt{b}}\right\}$$

$$\left\{\text{Int}\left[\frac{1}{4 + x^2}, x\right], \text{Int}\left[\frac{1}{4 - x^2}, x\right]\right\}$$

$$\left\{\frac{1}{2}\text{ArcTan}\left[\frac{x}{2}\right], \frac{1}{2}\text{ArcTanh}\left[\frac{x}{2}\right]\right\}$$

- *Mathematica* knows the symmetric rules but does not always use them:

$$\left\{\int \frac{1}{a + b x^2} \, dx, \int \frac{1}{a - b x^2} \, dx\right\}$$

$$\left\{\frac{\text{ArcTan}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{\sqrt{a}\,\sqrt{b}}, \frac{\text{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{\sqrt{a}\,\sqrt{b}}\right\}$$

$$\left\{\int \frac{1}{4 + x^2} \, dx, \int \frac{1}{4 - x^2} \, dx\right\}$$

$$\left\{\frac{1}{2}\text{ArcTan}\left[\frac{x}{2}\right], \frac{1}{4}\text{Log}[-2 - x] - \frac{1}{4}\text{Log}[-2 + x]\right\}$$

- *Maple* knows the symmetric rules but does not always use them:

```
[int(1/(a+b*x^2), x), int(1/(a-b*x^2), x)];
```

$$\left\{\frac{\text{ArcTan}\left[\frac{b\, x}{\sqrt{a\, b}}\right]}{\sqrt{a\, b}}, \frac{\text{ArcTanh}\left[\frac{b\, x}{\sqrt{a\, b}}\right]}{\sqrt{a\, b}}\right\}$$

```
[int(1/(4+x^2), x), int(1/(4-x^2), x)];
```

$$\left\{\frac{1}{2}\text{ArcTan}\left[\frac{x}{2}\right], -\frac{1}{4}\text{Log}[-2 + x] + \frac{1}{4}\text{Log}[2 + x]\right\}$$

Rational Function Integration Problem 8

$$\int \frac{1}{a + b\,x + c\,x^2}\,dx$$

- The *Rubi* results are symmetric:

```
Int[1/(3 + 5 x + 4 x^2), x]
```

$$\frac{2\,\text{ArcTan}\left[\frac{5+8\,x}{\sqrt{23}}\right]}{\sqrt{23}}$$

```
Int[1/(3 + 5 x - 4 x^2), x]
```

$$-\frac{2\,\text{ArcTanh}\left[\frac{5-8\,x}{\sqrt{73}}\right]}{\sqrt{73}}$$

- The *Mathematica* results are *not* symmetric:

$$\int \frac{1}{3 + 5\,x + 4\,x^2}\,dx$$

$$\frac{2\,\text{ArcTan}\left[\frac{5+8\,x}{\sqrt{23}}\right]}{\sqrt{23}}$$

$$\int \frac{1}{3 + 5\,x - 4\,x^2}\,dx$$

$$\frac{-\text{Log}\left[5+\sqrt{73}-8\,x\right]+\text{Log}\left[-5+\sqrt{73}+8\,x\right]}{\sqrt{73}}$$

- The *Maple* results are symmetric:

```
int(1/(3 + 5*x + 4*x^2), x);
```

$$\frac{2\,\text{ArcTan}\left[\frac{5+8\,x}{\sqrt{23}}\right]}{\sqrt{23}}$$

```
int(1/(3 + 5*x - 4*x^2), x);
```

$$\frac{2\,\text{ArcTanh}\left[\frac{-5+8\,x}{\sqrt{73}}\right]}{\sqrt{73}}$$

Rational Function Integration Problem 9

$$\int \frac{x^n}{1-x^6}\, dx$$

- The *Rubi* results are relatively simple 3 term sums:

```
Int[1/(1-x^6), x]
```

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,x}{1-x^2}\right]}{2\sqrt{3}}+\frac{\text{ArcTanh}[x]}{3}+\frac{1}{6}\,\text{ArcTanh}\left[\frac{x}{1+x^2}\right]$$

```
Int[x/(1-x^6), x]
```

$$\frac{\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{6}\,\text{Log}\left[1-x^2\right]+\frac{1}{12}\,\text{Log}\left[1+x^2+x^4\right]$$

```
Int[x^3/(1-x^6), x]
```

$$-\frac{\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{6}\,\text{Log}\left[1-x^2\right]+\frac{1}{12}\,\text{Log}\left[1+x^2+x^4\right]$$

```
Int[x^4/(1-x^6), x]
```

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,x}{1-x^2}\right]}{2\sqrt{3}}+\frac{\text{ArcTanh}[x]}{3}+\frac{1}{6}\,\text{ArcTanh}\left[\frac{x}{1+x^2}\right]$$

- The *Mathematica* results are more complicated 6 term sums:

$$\int \frac{1}{1-x^6}\, dx$$

$$\frac{1}{12}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]+2\sqrt{3}\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]-2\,\text{Log}[-1+x]+2\,\text{Log}[1+x]-\text{Log}\left[1-x+x^2\right]+\text{Log}\left[1+x+x^2\right]\right)$$

$$\int \frac{x}{1-x^6}\, dx$$

$$\frac{1}{12}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]-2\sqrt{3}\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]-2\,\text{Log}[-1+x]-2\,\text{Log}[1+x]+\text{Log}\left[1-x+x^2\right]+\text{Log}\left[1+x+x^2\right]\right)$$

$$\int \frac{x^3}{1-x^6}\, dx$$

$$\frac{1}{12}\left(-2\sqrt{3}\ \text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]+2\sqrt{3}\ \text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]-2\,\text{Log}[-1+x]-2\,\text{Log}[1+x]+\text{Log}\left[1-x+x^2\right]+\text{Log}\left[1+x+x^2\right]\right)$$

$$\int \frac{x^4}{1-x^6}\, dx$$

$$\frac{1}{12}\left(-2\sqrt{3}\ \text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]-2\sqrt{3}\ \text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]-2\,\text{Log}[-1+x]+2\,\text{Log}[1+x]-\text{Log}\left[1-x+x^2\right]+\text{Log}\left[1+x+x^2\right]\right)$$

- The *Maple* results are more complicated 6 term sums:

```
int(1/(1-x^6), x);
```

$$\frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{6}\text{Log}[-1+x]+\frac{1}{6}\text{Log}[1+x]-\frac{1}{12}\text{Log}\left[1-x+x^2\right]+\frac{1}{12}\text{Log}\left[1+x+x^2\right]$$

```
int(x/(1-x^6), x);
```

$$\frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{6}\text{Log}[-1+x]-\frac{1}{6}\text{Log}[1+x]+\frac{1}{12}\text{Log}\left[1-x+x^2\right]+\frac{1}{12}\text{Log}\left[1+x+x^2\right]$$

```
int(x^3/(1-x^6), x);
```

$$-\frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{6}\text{Log}[-1+x]-\frac{1}{6}\text{Log}[1+x]+\frac{1}{12}\text{Log}\left[1-x+x^2\right]+\frac{1}{12}\text{Log}\left[1+x+x^2\right]$$

```
int(x^4/(1-x^6), x);
```

$$-\frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{6}\text{Log}[-1+x]+\frac{1}{6}\text{Log}[1+x]-\frac{1}{12}\text{Log}\left[1-x+x^2\right]+\frac{1}{12}\text{Log}\left[1+x+x^2\right]$$

Rational Function Integration Problem 10

$$\int \frac{1}{1+x^8}\, dx$$

- The *Rubi* result is a 4 term sum:

```
Int[1/(1+x^8), x]
```

$$\frac{\left(-1+\sqrt{2}\right)\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}\,x}{1-x^2}\right]}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\left(1+\sqrt{2}\right)\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}\,x}{1-x^2}\right]}{4\sqrt{2\left(2+\sqrt{2}\right)}}+$$
$$\frac{\left(-1+\sqrt{2}\right)\text{ArcTanh}\left[\frac{\sqrt{2-\sqrt{2}}\,x}{1+x^2}\right]}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\left(1+\sqrt{2}\right)\text{ArcTanh}\left[\frac{\sqrt{2+\sqrt{2}}\,x}{1+x^2}\right]}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

- The *Mathematica* result is an 8 term sum:

$$\int \frac{1}{1+x^8}\, dx$$

$$\frac{1}{4}\text{ArcTan}\left[\text{Sec}\left[\frac{\pi}{8}\right]\left(x-\text{Sin}\left[\frac{\pi}{8}\right]\right)\right]\text{Cos}\left[\frac{\pi}{8}\right]+\frac{1}{4}\text{ArcTan}\left[\text{Sec}\left[\frac{\pi}{8}\right]\left(x+\text{Sin}\left[\frac{\pi}{8}\right]\right)\right]\text{Cos}\left[\frac{\pi}{8}\right]-$$
$$\frac{1}{8}\text{Cos}\left[\frac{\pi}{8}\right]\text{Log}\left[1+x^2-2\,x\,\text{Cos}\left[\frac{\pi}{8}\right]\right]+\frac{1}{8}\text{Cos}\left[\frac{\pi}{8}\right]\text{Log}\left[1+x^2+2\,x\,\text{Cos}\left[\frac{\pi}{8}\right]\right]+$$
$$\frac{1}{4}\text{ArcTan}\left[\left(x-\text{Cos}\left[\frac{\pi}{8}\right]\right)\text{Csc}\left[\frac{\pi}{8}\right]\right]\text{Sin}\left[\frac{\pi}{8}\right]+\frac{1}{4}\text{ArcTan}\left[\left(x+\text{Cos}\left[\frac{\pi}{8}\right]\right)\text{Csc}\left[\frac{\pi}{8}\right]\right]\text{Sin}\left[\frac{\pi}{8}\right]-$$
$$\frac{1}{8}\text{Log}\left[1+x^2-2\,x\,\text{Sin}\left[\frac{\pi}{8}\right]\right]\text{Sin}\left[\frac{\pi}{8}\right]+\frac{1}{8}\text{Log}\left[1+x^2+2\,x\,\text{Sin}\left[\frac{\pi}{8}\right]\right]\text{Sin}\left[\frac{\pi}{8}\right]$$

- The *Maple* result is *not* in closed-form:

```
int(1/(1+x^8), x);
```

```
sum(_R*ln(x+8*_R), _R = RootOf(16777216*_Z^8+1))
```

Rational Function Integration Problem 11

$$\int \frac{1}{1+x^2+x^4}\,dx$$

- The *Rubi* result is free of the imaginary unit:

```
Int[1/(1 + x^2 + x^4), x]
```

$$\frac{1}{2}\left(\frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{2}\text{Log}\left[1-x+x^2\right]\right)+\frac{1}{2}\left(\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}\text{Log}\left[1+x+x^2\right]\right)$$

- The *Mathematica* result is not free of the imaginary unit:

$$\int \frac{1}{1+x^2+x^4}\,dx$$

$$\frac{1}{6}\,i\left(\sqrt{6-6\,i\,\sqrt{3}}\;\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]-\sqrt{6+6\,i\,\sqrt{3}}\;\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]\right)$$

- The *Maple* result is free of the imaginary unit:

```
int(1/(1+x^2+x^4), x);
```

$$\frac{1}{2}\left(\frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{2}\text{Log}\left[1-x+x^2\right]\right)+\frac{1}{2}\left(\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}\text{Log}\left[1+x+x^2\right]\right)$$

Rational Function Integration Problem 12

$$\int \frac{1+x^2}{1+b\,x^2+x^4}\,dx$$

- The *Rubi* results are simple single terms:

```
Int[(1+x^2)/(1+b x^2+x^4), x]
```

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2+b}\,x}{1-x^2}\right]}{\sqrt{2+b}}$$

```
Int[(1+x^2)/(1+x^2+x^4), x]
```

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,x}{1-x^2}\right]}{\sqrt{3}}$$

- The *Mathematica* results are more complicated 2 term sums and can include the imaginary unit:

$$\int \frac{1+x^2}{1+b\,x^2+x^4}\,dx$$

$$\frac{\frac{\left(2-b+\sqrt{-4+b^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{b-\sqrt{-4+b^2}}}\right]}{\sqrt{b-\sqrt{-4+b^2}}}+\frac{\left(-2+b+\sqrt{-4+b^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{b+\sqrt{-4+b^2}}}\right]}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\,\sqrt{-4+b^2}}$$

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx$$

$$\frac{\left(i+\sqrt{3}\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]}{\sqrt{6+6\,i\,\sqrt{3}}}+\frac{\left(-i+\sqrt{3}\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]}{\sqrt{6-6\,i\,\sqrt{3}}}$$

- The *Maple* results are more complicated 2 term sums:

```
int((1+x^2)/(1+b*x^2+x^4), x);
```

$$\frac{\left(2-b+\sqrt{-4+b^2}\right)\sqrt{b+\sqrt{-4+b^2}}\;\text{ArcTan}\left[\frac{2\,x}{\sqrt{2\,b-2\,\sqrt{-4+b^2}}}\right]+\sqrt{b-\sqrt{-4+b^2}}\;\left(-2+b+\sqrt{-4+b^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{b+\sqrt{-4+b^2}}}\right]}{\sqrt{2}\,\sqrt{-4+b^2}\,\sqrt{b-\sqrt{-4+b^2}}\,\sqrt{b+\sqrt{-4+b^2}}}$$

```
int((1+x^2)/(1+x^2+x^4), x);
```

$$\frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{\sqrt{3}}$$

Rational Function Integration Problem 13

$$\int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx$$

- The *Rubi* result is in elementary form and simple:

$$\text{Int}\left[\frac{1}{x\left(1+x^5+x^{10}\right)},x\right]$$

$$-\frac{\text{ArcTan}\left[\frac{1+2x^5}{\sqrt{3}}\right]}{5\sqrt{3}}+\text{Log}[x]-\frac{1}{10}\text{Log}\left[1+x^5+x^{10}\right]$$

- The *Mathematica* result is not in closed-form:

$$\int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx$$

$$\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{5\sqrt{3}}+\text{Log}[x]-\frac{1}{10}\text{Log}\left[1+x+x^2\right]-\frac{1}{5}\text{RootSum}\Big[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7+\#1^8\ \&,$$
$$\Big(-\text{Log}[x-\#1]\,\#1+2\,\text{Log}[x-\#1]\,\#1^2-\text{Log}[x-\#1]\,\#1^3+3\,\text{Log}[x-\#1]\,\#1^4-\text{Log}[x-\#1]\,\#1^5-$$
$$3\,\text{Log}[x-\#1]\,\#1^6+4\,\text{Log}[x-\#1]\,\#1^7\Big)\Big/\left(-1+3\,\#1^2-4\,\#1^3+5\,\#1^4-7\,\#1^6+8\,\#1^7\right)\ \&\Big]$$

- The *Maple* result is in elementary form but not as simple:

```
int(1/(x*(1+x^5+x^10)), x);
```

$$-\frac{\text{ArcTan}\left[\frac{1+2x^5}{\sqrt{3}}\right]}{5\sqrt{3}}+\text{Log}[x]-\frac{1}{10}\text{Log}\left[1+x+x^2\right]-\frac{1}{10}\text{Log}\left[1-x+x^3-x^4+x^5-x^7+x^8\right]$$