Inverse Trig Function Integration Problem 1

$$\int \text{ArcSec}[a + b\,x]\,dx$$

- *Rubi* uses the substitution u=a+b x to generalize rule:

```
Int[ArcSec[x], x]
```

$$x\,\text{ArcSec}[x] - \text{ArcTanh}\left[\sqrt{1 - \frac{1}{x^2}}\right]$$

```
Int[ArcSec[a + b x], x]
```

$$\frac{(a + b\,x)\,\text{ArcSec}[a + b\,x]}{b} - \frac{\text{ArcTanh}\left[\sqrt{1 - \frac{1}{(a+b\,x)^2}}\right]}{b}$$

- *Mathematica* does *not* use the substitution u=a+b x to generalize rule:

$$\int \text{ArcSec}[x]\,dx$$

$$x\,\text{ArcSec}[x] - \frac{\sqrt{-1 + x^2}\,\text{Log}\left[2\left(x + \sqrt{-1 + x^2}\right)\right]}{\sqrt{1 - \frac{1}{x^2}}\,x}$$

$$\int \text{ArcSec}[a + b\,x]\,dx$$

$$x\,\text{ArcSec}[a + b\,x] - \frac{(a + b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\left(a\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+a^2+2\,a\,b\,x+b^2\,x^2}}\right] + \text{Log}\left[2\left(a + b\,x + \sqrt{-1 + a^2 + 2\,a\,b\,x + b^2\,x^2}\right)\right]\right)}{b\sqrt{-1 + a^2 + 2\,a\,b\,x + b^2\,x^2}}$$

- *Maple* uses the substitution u=a+b x to generalize rule:

```
int(arcsec(x), x);
```

$$x\,\text{ArcSec}[x] - \text{Log}\left[x + \sqrt{1 - \frac{1}{x^2}}\,x\right]$$

```
int(arcsec(a + b * x), x);
```

$$\frac{a\,\text{ArcSec}[a + b\,x]}{b} + x\,\text{ArcSec}[a + b\,x] - \frac{\text{Log}\left[a + b\,x + (a + b\,x)\sqrt{1 - \frac{1}{(a+b\,x)^2}}\right]}{b}$$

Note that these systems give similar results to the above for the arccosecant function.

Inverse Trig Function Integration Problem 2

$$\int \frac{\text{ArcSec}[a\,x^n]}{x}\,dx$$

- *Rubi* knows and takes advantage of the general rule for arbitrary n:

```
Int[ArcSec[a x^n]/x, x]
```

$$\frac{i\,\text{ArcSec}[a\,x^n]^2}{2\,n}-\frac{\text{ArcSec}[a\,x^n]\,\text{Log}\left[1-\frac{1}{\left(\frac{i\,x^{-n}}{a}+\sqrt{1-\frac{x^{-2\,n}}{a^2}}\right)^2}\right]}{n}+\frac{i\,\text{PolyLog}\left[2,\frac{1}{\left(\frac{i\,x^{-n}}{a}+\sqrt{1-\frac{x^{-2\,n}}{a^2}}\right)^2}\right]}{2\,n}$$

```
Int[ArcSec[a x^5]/x, x]
```

$$\frac{1}{10}\,i\,\text{ArcSec}[a\,x^5]^2-\frac{1}{5}\,\text{ArcSec}[a\,x^5]\,\text{Log}\left[1-\frac{1}{\left(\sqrt{1-\frac{1}{a^2\,x^{10}}}+\frac{i}{a\,x^5}\right)^2}\right]+\frac{1}{10}\,i\,\text{PolyLog}\left[2,\frac{1}{\left(\sqrt{1-\frac{1}{a^2\,x^{10}}}+\frac{i}{a\,x^5}\right)^2}\right]$$

- *Mathematica* does not know the elementary form of the general rule, but returns an elementary form when n is numeric:

$$\int \frac{\text{ArcSec}[a\,x^n]}{x}\,dx$$

$$\frac{x^{-n}\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},\frac{x^{-2\,n}}{a^2}\right]}{a\,n}+\left(\text{ArcSec}[a\,x^n]+\text{ArcSin}\left[\frac{x^{-n}}{a}\right]\right)\text{Log}[x]$$

$$\int \frac{\text{ArcSec}[a\,x^5]}{x}\,dx$$

$$\frac{1}{10}\,i\,\text{ArcSec}[a\,x^5]^2-\frac{1}{5}\,\text{ArcSec}[a\,x^5]\,\text{Log}\left[1+e^{2\,i\,\text{ArcSec}[a\,x^5]}\right]+\frac{1}{10}\,i\,\text{PolyLog}\left[2,-e^{2\,i\,\text{ArcSec}[a\,x^5]}\right]$$

- *Maple* knows the general rule for arbitary n, but does not use it when n is numeric:

```
int(arcsec(a*x^n)/x, x);
```

$$\frac{i\,\text{ArcSec}[a\,x^n]^2}{2\,n}-\frac{\text{ArcSec}[a\,x^n]\,\text{Log}\left[1+\left(\frac{x^{-n}}{a}+i\,\sqrt{1-\frac{x^{-2\,n}}{a^2}}\right)^2\right]}{n}+\frac{i\,\text{PolyLog}\left[2,-\left(\frac{x^{-n}}{a}+i\,\sqrt{1-\frac{x^{-2\,n}}{a^2}}\right)^2\right]}{2\,n}$$

```
int(arcsec(a*x^5)/x, x);
```

$$\int \frac{\text{ArcSec}[a\,x^5]}{x}\,dx$$

Note that these systems give similar results to the above for the arccosecant function.

Inverse Trig Function Integration Problem 3

$$\int \frac{\text{ArcTan}[x]}{\left(a+b x^2\right)^{m/2}} \, dx$$

- *Rubi* returns a relatively simple sum of (m+1)/2 terms for odd m>1:

$$\text{Int}\left[\frac{\text{ArcTan}[x]}{\left(a+b x^2\right)^{3/2}}, x\right]$$

$$\frac{x \, \text{ArcTan}[x]}{a \sqrt{a+b x^2}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a-b}}\right]}{a \sqrt{a-b}}$$

$$\text{Int}\left[\frac{\text{ArcTan}[x]}{\left(a+b x^2\right)^{5/2}}, x\right]$$

$$-\frac{1}{3 a (a-b) \sqrt{a+b x^2}}+\frac{x \left(3 a+2 b x^2\right) \text{ArcTan}[x]}{3 a^2 \left(a+b x^2\right)^{3/2}}+\frac{(3 a-2 b) \, \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a-b}}\right]}{3 a^2 (a-b)^{3/2}}$$

- *Mathematica* returns more complicated sums involving the imaginary unit:

$$\int \frac{\text{ArcTan}[x]}{\left(a+b x^2\right)^{3/2}} \, dx$$

$$\frac{x \, \text{ArcTan}[x]}{a \sqrt{a+b x^2}}+\frac{\text{Log}\left[-\frac{4 a \left(a-i b x+\sqrt{a-b} \sqrt{a+b x^2}\right)}{\sqrt{a-b} (i+x)}\right]}{2 a \sqrt{a-b}}+\frac{\text{Log}\left[-\frac{4 a \left(a+i b x+\sqrt{a-b} \sqrt{a+b x^2}\right)}{\sqrt{a-b} (-i+x)}\right]}{2 a \sqrt{a-b}}$$

$$\int \frac{\text{ArcTan}[x]}{\left(a+b x^2\right)^{5/2}} \, dx$$

$$-\frac{1}{3 a (a-b) \sqrt{a+b x^2}}+\frac{x \left(3 a+2 b x^2\right) \text{ArcTan}[x]}{3 a^2 \left(a+b x^2\right)^{3/2}}+$$

$$\frac{(3 a-2 b) \, \text{Log}\left[-\frac{12 a^2 \sqrt{a-b} \left(a-i b x+\sqrt{a-b} \sqrt{a+b x^2}\right)}{(3 a-2 b) (i+x)}\right]}{6 a^2 (a-b)^{3/2}}+\frac{(3 a-2 b) \, \text{Log}\left[-\frac{12 a^2 \sqrt{a-b} \left(a+i b x+\sqrt{a-b} \sqrt{a+b x^2}\right)}{(3 a-2 b) (-i+x)}\right]}{6 a^2 (a-b)^{3/2}}$$

- *Maple* is unable to integrate the expressions:

```
int(arctan(x)/(a+b*x^2)^(3/2), x);
```

$$\int \frac{\text{ArcTan}[x]}{\left(a+b x^2\right)^{3/2}} \, dx$$

```
int (arctan (x) / (a + b * x^2) ^ (5 / 2), x);
```

$$\int \frac{\text{ArcTan}[x]}{\left(a + b\,x^2\right)^{5/2}}\,dx$$

Note that these systems give similar results to the above for the arccotangent, hyperbolic arctangent and hyperbolic arccotangent functions.

Inverse Trig Function Integration Problem 4

$$\int \mathrm{ArcTan}\left[e^{a+b\,x}\right] dx$$

- The *Rubi* result is a relatively simple 2 term sum:

```
Int[ArcTan[e^(a+b x)], x]
```

$$\frac{i\,\mathrm{PolyLog}\left[2,-i\,e^{a+b\,x}\right]}{2\,b}-\frac{i\,\mathrm{PolyLog}\left[2,i\,e^{a+b\,x}\right]}{2\,b}$$

- The *Mathematica* result is a 5 term sum, the first 3 of which are superfluous since their derivative is 0:

$$\int \mathrm{ArcTan}\left[e^{a+b\,x}\right] dx$$

$$x\,\mathrm{ArcTan}\left[e^{a+b\,x}\right]-\frac{1}{2}\,i\,x\,\mathrm{Log}\left[1-i\,e^{a+b\,x}\right]+\frac{1}{2}\,i\,x\,\mathrm{Log}\left[1+i\,e^{a+b\,x}\right]+\frac{i\,\mathrm{PolyLog}\left[2,-i\,e^{a+b\,x}\right]}{2\,b}-\frac{i\,\mathrm{PolyLog}\left[2,i\,e^{a+b\,x}\right]}{2\,b}$$

- The *Maple* result is a complicated 6 term sum:

```
int(arctan(exp(a+b*x)), x);
```

$$-\frac{\mathrm{ArcTan}\left[e^{a+b\,x}\right]\mathrm{Log}\left[\frac{2\,i}{i+e^{a+b\,x}}\right]}{b}+\frac{\mathrm{ArcTan}\left[e^{a+b\,x}\right]\mathrm{Log}\left[1-\frac{1+i\,e^{a+b\,x}}{\sqrt{1+e^{2\,(a+b\,x)}}}\right]}{b}+\frac{\mathrm{ArcTan}\left[e^{a+b\,x}\right]\mathrm{Log}\left[1+\frac{1+i\,e^{a+b\,x}}{\sqrt{1+e^{2\,(a+b\,x)}}}\right]}{b}+$$

$$\frac{i\,\mathrm{PolyLog}\left[2,\frac{-i+e^{a+b\,x}}{i+e^{a+b\,x}}\right]}{2\,b}-\frac{i\,\mathrm{PolyLog}\left[2,-\frac{1+i\,e^{a+b\,x}}{\sqrt{1+e^{2\,(a+b\,x)}}}\right]}{b}-\frac{i\,\mathrm{PolyLog}\left[2,\frac{1+i\,e^{a+b\,x}}{\sqrt{1+e^{2\,(a+b\,x)}}}\right]}{b}$$

Note that these systems give similar results to the above for the arccotangent function.