Algebraic Function Integration Problem 1

$$\int (x^n)^{-1/n}\,dx$$

- *Rubi* knows and takes advantage of the general rule:

```
Int[(x^n)^(-1/n), x]
```

$x\,(x^n)^{-1/n}\,\mathrm{Log}[x]$

```
Int[(x^3)^(-1/3), x]
```

$\frac{x\,\mathrm{Log}[x]}{(x^3)^{1/3}}$

- *Mathematica* knows and takes advantage of the general rule:

$$\int (x^n)^{-1/n}\,dx$$

$x\,(x^n)^{-1/n}\,\mathrm{Log}[x]$

$$\int (x^3)^{-1/3}\,dx$$

$\frac{x\,\mathrm{Log}[x]}{(x^3)^{1/3}}$

- *Maple* knows but does *not* take advantage of the general rule and gets an incorrect result:

```
int((x^n)^(-1/n), x);
```

$x\,(x^n)^{-1/n}\,\mathrm{Log}[x]$

```
int((x^3)^(-1/3), x);
```

Log[x]

Algebraic Function Integration Problem 2

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx$$

- *Rubi* knows and takes advantage of the general rule:

$$\text{Int}\left[\frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}},x\right]$$

$$\frac{2\left(a+b\sqrt{x}\right)^{1+n}}{b\,(1+n)}$$

$$\text{Int}\left[\frac{\left(a+b\sqrt{x}\right)^{20}}{\sqrt{x}},x\right]$$

$$\frac{2\left(a+b\sqrt{x}\right)^{21}}{21\,b}$$

- *Mathematica* knows but does *not* take advantage of the general rule:

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx$$

$$\frac{2\left(a+b\sqrt{x}\right)^{1+n}}{b\,(1+n)}$$

$$\int \frac{\left(a+b\sqrt{x}\right)^{20}}{\sqrt{x}}\,dx$$

$$2a^{20}\sqrt{x}+20a^{19}bx+\frac{380}{3}a^{18}b^2x^{3/2}+570a^{17}b^3x^2+1938a^{16}b^4x^{5/2}+$$
$$5168a^{15}b^5x^3+\frac{77\,520}{7}a^{14}b^6x^{7/2}+19\,380a^{13}b^7x^4+\frac{83\,980}{3}a^{12}b^8x^{9/2}+33\,592a^{11}b^9x^5+$$
$$33\,592a^{10}b^{10}x^{11/2}+\frac{83\,980}{3}a^9b^{11}x^6+19\,380a^8b^{12}x^{13/2}+\frac{77\,520}{7}a^7b^{13}x^7+5168a^6b^{14}x^{15/2}+$$
$$1938a^5b^{15}x^8+570a^4b^{16}x^{17/2}+\frac{380}{3}a^3b^{17}x^9+20a^2b^{18}x^{19/2}+2ab^{19}x^{10}+\frac{2}{21}b^{20}x^{21/2}$$

- *Maple* knows but does *not* take advantage of the general rule:

```
int((a+b*sqrt(x))^n/sqrt(x), x);
```

$$\frac{2\left(a+b\sqrt{x}\right)^{1+n}}{b\,(1+n)}$$

```
int((a+b*sqrt(x))^20/sqrt(x), x);
```

$$2 a^{20} \sqrt{x} + 20 a^{19} b x + \frac{380}{3} a^{18} b^2 x^{3/2} + 570 a^{17} b^3 x^2 + 1938 a^{16} b^4 x^{5/2} + 5168 a^{15} b^5 x^3 + \frac{77520}{7} a^{14} b^6 x^{7/2} + 19380 a^{13} b^7 x^4 + \frac{83980}{3} a^{12} b^8 x^{9/2} + 33592 a^{11} b^9 x^5 + 33592 a^{10} b^{10} x^{11/2} + \frac{83980}{3} a^9 b^{11} x^6 + 19380 a^8 b^{12} x^{13/2} + \frac{77520}{7} a^7 b^{13} x^7 + 5168 a^6 b^{14} x^{15/2} + 1938 a^5 b^{15} x^8 + 570 a^4 b^{16} x^{17/2} + \frac{380}{3} a^3 b^{17} x^9 + 20 a^2 b^{18} x^{19/2} + 2 a b^{19} x^{10} + \frac{2}{21} b^{20} x^{21/2}$$

Algebraic Function Integration Problem 3

$$\int \sqrt{\frac{a + b\,x^n}{x^2}}\,dx$$

- *Rubi* knows and always takes advantage of the general rule:

$$\text{Int}\left[\sqrt{\frac{a + b\,x^n}{x^2}}, x\right]$$

$$\frac{2\,x\,\sqrt{\frac{a+b\,x^n}{x^2}}}{n} - \frac{2\,\sqrt{a}\,x\,\sqrt{\frac{a+b\,x^n}{x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right]}{n\,\sqrt{a + b\,x^n}}$$

$$\text{Int}\left[\sqrt{\frac{a + b\,x^4}{x^2}}, x\right]$$

$$\frac{1}{2}\,x\,\sqrt{\frac{a + b\,x^4}{x^2}} - \frac{\sqrt{a}\,x\,\sqrt{\frac{a+b\,x^4}{x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{2\,\sqrt{a + b\,x^4}}$$

$$\text{Int}\left[\sqrt{\frac{a + b\,x^5}{x^2}}, x\right]$$

$$\frac{2}{5}\,x\,\sqrt{\frac{a + b\,x^5}{x^2}} - \frac{2\,\sqrt{a}\,x\,\sqrt{\frac{a+b\,x^5}{x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^5}}{\sqrt{a}}\right]}{5\,\sqrt{a + b\,x^5}}$$

- *Mathematica* knows but does *not* always take advantage of the general rule:

$$\int \sqrt{\frac{a + b\,x^n}{x^2}}\,dx$$

$$\frac{2\,x\,\sqrt{\frac{a+b\,x^n}{x^2}}}{n} - \frac{2\,\sqrt{a}\,x\,\sqrt{\frac{a+b\,x^n}{x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right]}{n\,\sqrt{a + b\,x^n}}$$

$$\int \sqrt{\frac{a + b\,x^4}{x^2}}\,dx$$

$$\frac{1}{2}\,x\,\sqrt{\frac{a + b\,x^4}{x^2}} + \frac{\sqrt{a}\,x\,\sqrt{\frac{a+b\,x^4}{x^2}}\,\left(\text{Log}\left[x^2\right] - \text{Log}\left[a + \sqrt{a}\,\sqrt{a + b\,x^4}\right]\right)}{2\,\sqrt{a + b\,x^4}}$$

$$\int \sqrt{\frac{a + b x^5}{x^2}} \, dx$$

$$\frac{2}{5} x \sqrt{\frac{a + b x^5}{x^2}} - \frac{2 \sqrt{a} \, x \sqrt{\frac{a+b x^5}{x^2}} \, \text{ArcTanh}\left[\frac{\sqrt{a+b x^5}}{\sqrt{a}}\right]}{5 \sqrt{a + b x^5}}$$

- *Maple* knows but does *not* take advantage of the general rule:

```
int(sqrt((a+b*x^n)/x^2), x);
```

$$\frac{2 x \sqrt{\frac{a+b x^n}{x^2}}}{n} - \frac{2 \sqrt{a} \, x \sqrt{\frac{a+b x^n}{x^2}} \, \text{ArcTanh}\left[\frac{\sqrt{a+b x^n}}{\sqrt{a}}\right]}{n \sqrt{a + b x^n}}$$

```
int(sqrt((a+b*x^4)/x^2), x);
```

$$\frac{1}{2} x \sqrt{\frac{a + b x^4}{x^2}} + \frac{\sqrt{a} \, x \sqrt{\frac{a+b x^4}{x^2}} \, \text{Log}[2]}{2 \sqrt{a + b x^4}} - \frac{\sqrt{a} \, x \sqrt{\frac{a+b x^4}{x^2}} \, \text{Log}\left[\frac{a+\sqrt{a} \sqrt{a+b x^4}}{x^2}\right]}{2 \sqrt{a + b x^4}}$$

```
int(sqrt((a+b*x^5)/x^2), x);
```

$$\int \sqrt{\frac{a + b x^5}{x^2}} \, dx$$

Algebraic Function Integration Problem 4

$$\int 1 \Big/ \sqrt{\frac{a + b x^n}{x^{n-2}}} \, dx$$

- *Rubi* knows and takes advantage of the general rule:

```
Int[1/Sqrt[(a+b*x^n)/x^(n-2)], x]
```

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{x^{2-n}\,(a+b x^n)}}\right]}{\sqrt{b}\, n}$$

```
Int[1/Sqrt[(a+b*x^5)/x^3], x]
```

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{\frac{a+b x^5}{x^3}}}\right]}{5 \sqrt{b}}$$

- *Mathematica* knows but does *not* take advantage of the general rule:

$$\int 1 \Big/ \sqrt{\frac{a + b x^n}{x^{n-2}}} \, dx$$

$$\frac{2 x^{1-\frac{n}{2}} \sqrt{a + b x^n}\, \operatorname{Log}\left[b x^{n/2} + \sqrt{b} \sqrt{a + b x^n}\right]}{\sqrt{b}\, n \sqrt{x^2 \,(b + a x^{-n})}}$$

$$\int 1 \Big/ \sqrt{\frac{a + b x^5}{x^3}} \, dx$$

$$\int \frac{1}{\sqrt{\frac{a+b x^5}{x^3}}} \, dx$$

- *Maple* is unable to integrate either expression:

```
int(1/sqrt((a+b*x^n)/x^(n-2)), x);
```

$$\int \frac{1}{\sqrt{\frac{a+b x^n}{x^{n-2}}}} \, dx$$

```
int((a+b*sqrt(x))^20/sqrt(x), x);
```

$$\int \frac{1}{\sqrt{\frac{a+b\,x^5}{x^3}}}\,dx$$

Algebraic Function Integration Problem 5

$$\int \frac{1}{\sqrt{1+x}\,\sqrt{-1+x}}\,dx$$

- The *Rubi* result is simple:

```
Int[1/(Sqrt[1+x] Sqrt[-1+x]), x]
```

$\text{ArcCosh}[x]$

- The *Mathematica* result is more complicated:

$$\int \frac{1}{\sqrt{1+x}\,\sqrt{-1+x}}\,dx$$

$2\,\text{ArcSinh}\left[\frac{\sqrt{-1+x}}{\sqrt{2}}\right]$

- The *Maple* result is even more complicated:

```
int(1/(sqrt(1+x)*sqrt(-1+x)), x);
```

$$\frac{\sqrt{(-1+x)\,(1+x)}\;\text{Log}\left[x+\sqrt{-1+x^2}\right]}{\sqrt{-1+x}\,\sqrt{1+x}}$$

Algebraic Function Integration Problem 6

$$\int \frac{1}{\sqrt{a+b x}\,\sqrt{c+d x}}\,dx$$

- The *Rubi* results are simple and symmetric:

```
Int[1/(Sqrt[a+b x] Sqrt[c+d x]), x]
```

$$\frac{2\,\mathrm{ArcTanh}\left[\frac{\sqrt{d}\,\sqrt{a+b x}}{\sqrt{b}\,\sqrt{c+d x}}\right]}{\sqrt{b}\,\sqrt{d}}$$

```
Int[1/(Sqrt[a-b x] Sqrt[c+d x]), x]
```

$$-\frac{2\,\mathrm{ArcTan}\left[\frac{\sqrt{d}\,\sqrt{a-b x}}{\sqrt{b}\,\sqrt{c+d x}}\right]}{\sqrt{b}\,\sqrt{d}}$$

- The *Mathematica* results are more complicated and involve the imaginary unit:

$$\int \frac{1}{\sqrt{a+b x}\,\sqrt{c+d x}}\,dx$$

$$\frac{\mathrm{Log}\left[b c+a d+2 b d x+2\sqrt{b}\,\sqrt{d}\,\sqrt{a+b x}\,\sqrt{c+d x}\right]}{\sqrt{b}\,\sqrt{d}}$$

$$\int \frac{1}{\sqrt{a-b x}\,\sqrt{c+d x}}\,dx$$

$$\frac{i\,\mathrm{Log}\left[2\sqrt{a-b x}\,\sqrt{c+d x}-\frac{i\,(b c-a d+2 b d x)}{\sqrt{b}\,\sqrt{d}}\right]}{\sqrt{b}\,\sqrt{d}}$$

- The *Maple* results are more complicated and not symmetric:

```
int(1/(sqrt(a+b*x)*sqrt(c+d*x)), x);
```

$$\frac{\sqrt{(a+b*x)\,(c+d*x)}\;\mathrm{Log}\left[\frac{\frac{d a}{2}+\frac{b c}{2}+b d x}{\sqrt{b d}}+\sqrt{a c+(a d+b c)\,x+b d x^2}\right]}{\sqrt{a+b x}\,\sqrt{c+d x}\,\sqrt{b d}}$$

```
int(1/(sqrt(a-b*x)*sqrt(c+d*x)), x);
```

$$\frac{\sqrt{(a-b*x)\,(c+d*x)}\;\mathrm{ArcTan}\left[\frac{\sqrt{b d}\,\left(x-\frac{a d-b c}{2 b d}\right)}{\sqrt{a c+(a d-b c)\,x-b d x^2}}\right]}{\sqrt{a-b x}\,\sqrt{c+d x}\,\sqrt{b d}}$$

Algebraic Function Integration Problem 7

$$\int \frac{1}{(a+b\,x)^{1/4}\,(c+d\,x)^{3/4}}\,dx$$

- The *Rubi* results involves only elementary functions:

$$\text{Int}\left[\frac{1}{(a+b\,x)^{1/4}\,(c+d\,x)^{3/4}},\,x\right]$$

$$-\frac{2\,\text{ArcTan}\left[\frac{d^{1/4}\,(a+b\,x)^{1/4}}{b^{1/4}\,(c+d\,x)^{1/4}}\right]}{b^{1/4}\,d^{3/4}}+\frac{2\,\text{ArcTanh}\left[\frac{d^{1/4}\,(a+b\,x)^{1/4}}{b^{1/4}\,(c+d\,x)^{1/4}}\right]}{b^{1/4}\,d^{3/4}}$$

- The *Mathematica* result involves *non*elementary functions:

$$\int \frac{1}{(a+b\,x)^{1/4}\,(c+d\,x)^{3/4}}\,dx$$

$$\frac{4\,\left(\frac{d\,(a+b\,x)}{-b\,c+a\,d}\right)^{1/4}\,(c+d\,x)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{1}{4},\,\frac{5}{4},\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{d\,(a+b\,x)^{1/4}}$$

- *Maple* is unable to integrate the expression:

```
int(1/((a+b*x)^(1/4)*(c+d*x)^(3/4)),x);
```

$$\int \frac{1}{(a+b\,x)^{1/4}\,(c+d\,x)^{3/4}}\,dx$$

Algebraic Function Integration Problem 8

$$\int \frac{1}{\sqrt{(a+x)\,(b+x)}}\,dx$$

- The *Rubi* results are simple and symmetric:

```
Int[1/Sqrt[(a + x) (b + x)], x]
```

$$\mathrm{ArcTanh}\left[\frac{a+b+2x}{2\sqrt{a\,b+(a+b)\,x+x^2}}\right]$$

```
Int[1/Sqrt[(a - x) (b + x)], x]
```

$$-\mathrm{ArcTan}\left[\frac{a-b-2x}{2\sqrt{a\,b+(a-b)\,x-x^2}}\right]$$

- The *Mathematica* results are more complicated and not symmetric:

$$\int \frac{1}{\sqrt{(a+x)\,(b+x)}}\,dx$$

$$\frac{\sqrt{a+x}\,\sqrt{b+x}\,\mathrm{Log}\left[a+b+2x+2\sqrt{a+x}\,\sqrt{b+x}\right]}{\sqrt{(a+x)\,(b+x)}}$$

$$\int \frac{1}{\sqrt{(a-x)\,(b+x)}}\,dx$$

$$-\frac{\sqrt{a-x}\,\sqrt{b+x}\,\mathrm{ArcTan}\left[\frac{a-b-2x}{2\sqrt{a-x}\,\sqrt{b+x}}\right]}{\sqrt{(a-x)\,(b+x)}}$$

- The *Maple* results are simple but not symmetric:

```
int(1/sqrt((a + x) * (b + x)), x);
```

$$\mathrm{Log}\left[\frac{a}{2}+\frac{b}{2}+x+\sqrt{a\,b+(a+b)\,x+x^2}\right]$$

```
int(1/sqrt((a - x) * (b + x)), x);
```

$$-\mathrm{ArcTan}\left[\frac{a-b-2x}{2\sqrt{a\,b+(a-b)\,x-x^2}}\right]$$

Algebraic Function Integration Problem 9

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx$$

- The *Rubi* result is the sum of 3 simple terms:

```
Int[(Sqrt[1-x]+Sqrt[1+x])^2/x, x]
```

$$2\sqrt{1-x}\,\sqrt{1+x}-4\,\text{ArcTanh}\left[\frac{\sqrt{1-x}}{\sqrt{1+x}}\right]+2\,\text{Log}[x]$$

- The *Mathematica* result is the sum of 6 simple terms:

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx$$

$$2\sqrt{1-x^2}+2\,\text{Log}[x]-2\,\text{Log}\left[2+\sqrt{1-x}-\sqrt{1+x}\right]+ 2\,\text{Log}\left[-1+\sqrt{1+x}\right]-2\,\text{Log}\left[1+\sqrt{1+x}\right]+2\,\text{Log}\left[2+\sqrt{1-x}+\sqrt{1+x}\right]$$

- The *Maple* result is the sum of 3 terms:

```
int((sqrt(1-x)+sqrt(1+x))^2/x, x);
```

$$2\sqrt{1-x}\,\sqrt{1+x}-\frac{2\sqrt{1-x}\,\sqrt{1+x}\,\text{ArcTanh}\left[\frac{1}{\sqrt{1-x^2}}\right]}{\sqrt{1-x^2}}+2\,\text{Log}[x]$$

Algebraic Function Integration Problem 10

$$\int \frac{1}{x \sqrt{1 + 1/x^n}} \, dx$$

- *Rubi* is consistent and able to integrate all the expressions:

$$\left\{\text{Int}\left[\frac{1}{x \sqrt{1+x^n}}, x\right], \text{Int}\left[\frac{1}{x \sqrt{1+x^5}}, x\right]\right\}$$

$$\left\{-\frac{2 \,\text{ArcTanh}\left[\sqrt{1+x^n}\right]}{n}, -\frac{2}{5} \text{ArcTanh}\left[\sqrt{1+x^5}\right]\right\}$$

$$\left\{\text{Int}\left[\frac{1}{x \sqrt{1+1/x^n}}, x\right], \text{Int}\left[\frac{1}{x \sqrt{1+1/x^5}}, x\right]\right\}$$

$$\left\{\frac{2 \,\text{ArcTanh}\left[\sqrt{1+x^{-n}}\right]}{n}, \frac{2}{5} \text{ArcTanh}\left[\sqrt{1+\frac{1}{x^5}}\right]\right\}$$

- *Mathematica* is unable to integrate a special case:

$$\left\{\int \frac{1}{x \sqrt{1+x^n}} \, dx, \int \frac{1}{x \sqrt{1+x^5}} \, dx\right\}$$

$$\left\{-\frac{2 \,\text{ArcTanh}\left[\sqrt{1+x^n}\right]}{n}, -\frac{2}{5} \text{ArcTanh}\left[\sqrt{1+x^5}\right]\right\}$$

$$\left\{\int \frac{1}{x \sqrt{1+1/x^n}} \, dx, \int \frac{1}{x \sqrt{1+1/x^5}} \, dx\right\}$$

$$\left\{\frac{2 \,\text{ArcTanh}\left[\sqrt{1+x^{-n}}\right]}{n}, \int \frac{1}{\sqrt{1+\frac{1}{x^5}}\; x} \, dx\right\}$$

- *Maple* is *not* consistent and unable to integrate all the expressions:

```
[int(1/(x*sqrt(1+x^n)),x), int(1/(x*sqrt(1+x^5)),x)]
```

$$\left\{-\frac{2 \,\text{ArcTanh}\left[\sqrt{1+x^n}\right]}{n}, \frac{2}{5} \text{Log}\left[\frac{-1+\sqrt{1+x^5}}{\sqrt{x^5}}\right]\right\}$$

```
[int(1/(x*sqrt(1+1/x^n)),x), int(1/(x*sqrt(1+1/x^5)),x)]
```

$$\left\{\frac{x^n \sqrt{x^{-n}\,(1+x^n)}\ \mathrm{Log}\left[\frac{1}{2}+x^n+\sqrt{x^n+x^{2n}}\right]}{n\sqrt{x^n\,(1+x^n)}},\ \int\frac{1}{\sqrt{1+\frac{1}{x^5}}\ x}\,dx\right\}$$

Algebraic Function Integration Problem 11

$$\int \frac{x}{x+\sqrt{x^6}}\, dx$$

- The *Rubi* result is free of nested square-roots

```
Simplify[Int[x / (x + Sqrt[x^6]), x]]
```

$$\frac{\left(x^3+\sqrt{x^6}\right) \text{ArcTan}[x]+\left(x^3-\sqrt{x^6}\right) \text{ArcTanh}[x]}{2\, x^3}$$

- *Mathematica* is unable to integrate the expression:

$$\int \frac{x}{x+\sqrt{x^6}}\, dx$$

$$\int \frac{x}{x+\sqrt{x^6}}\, dx$$

- The *Maple* result is simpler but *not* free of nested-square-roots:

```
int(x / (x + sqrt(x^6)), x);
```

$$\frac{\text{ArcTan}\left[x \sqrt{\frac{\sqrt{x^6}}{x^3}}\right]}{\sqrt{\frac{\sqrt{x^6}}{x^3}}}$$

Algebraic Function Integration Problem 12

$$\int \frac{\sqrt{1-x^2}}{1+x} \, dx$$

- The *Rubi* result is a simple sum:

```
Int[Sqrt[1 - x^2]/(1 + x), x]
```

$$\sqrt{1-x^2} + \text{ArcSin}[x]$$

- The *Mathematica* result is a more complicated sum involving a logarithm:

$$\int \frac{\sqrt{1-x^2}}{1+x} \, dx$$

$$\sqrt{1-x^2} - \frac{2\sqrt{1-x^2}\,\text{Log}\left[\sqrt{-1+x}+\sqrt{1+x}\right]}{\sqrt{-1+x}\,\sqrt{1+x}}$$

- The *Maple* result is a simple sum:

```
int(sqrt(1 - x^2) / (1 + x), x);
```

$$\sqrt{1-x^2} + \text{ArcSin}[x]$$

Algebraic Function Integration Problem 13

$$\int \frac{1}{x\,(a+b\,x)^{1/3}}\,dx$$

- The *Rubi* result involves only elementary functions:

```
Int[1/(x (a + b x)^(1/3)), x]
```

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{2\left(\frac{1}{2}+\frac{(a+b\,x)^{1/3}}{a^{1/3}}\right)}{\sqrt{3}}\right]+\text{Log}\left[-a^{1/3}+(a+b\,x)^{1/3}\right]-\frac{1}{2}\,\text{Log}\left[a^{2/3}+a^{1/3}\,(a+b\,x)^{1/3}+(a+b\,x)^{2/3}\right]}{a^{1/3}}$$

- The *Mathematica* result involves *non*elementary functions:

$$\int \frac{1}{x\,(a+b\,x)^{1/3}}\,dx$$

$$-\frac{3\left(\frac{a+b\,x}{b\,x}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{a}{b\,x}\right]}{(a+b\,x)^{1/3}}$$

- The *Maple* result involves only elementary functions:

```
int(1/(x*(a+b*x)^(1/3)), x);
```

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{2\left(\frac{1}{2}+\frac{(a+b\,x)^{1/3}}{a^{1/3}}\right)}{\sqrt{3}}\right]+\text{Log}\left[-a^{1/3}+(a+b\,x)^{1/3}\right]-\frac{1}{2}\,\text{Log}\left[a^{2/3}+a^{1/3}\,(a+b\,x)^{1/3}+(a+b\,x)^{2/3}\right]}{a^{1/3}}$$

Algebraic Function Integration Problem 14

$$\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx$$

- The *Rubi* result involves only elementary functions:

$$\text{Int}\left[\frac{x}{\left(1-x^3\right)^{2/3}},\,x\right]$$

$$\frac{\text{ArcTan}\left[\frac{-1+\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{6}\,\text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{1}{3}\,\text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]$$

- The *Mathematica* result involves *non*elementary functions:

$$\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx$$

$$\frac{1}{2}\,x^2\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,\frac{2}{3},\,\frac{5}{3},\,x^3\right]$$

- The *Maple* result involves *non*elementary functions:

```
int(x/(1-x^3)^(2/3),x);
```

$$\frac{1}{2}\,x^2\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,\frac{2}{3},\,\frac{5}{3},\,x^3\right]$$

Algebraic Function Integration Problem 15

$$\int \frac{x^{n/2}}{\sqrt{1+x^5}}\, dx$$

- *Rubi* is able to integrate the expression for n mod 10 = 3:

$$\text{Int}\left[\frac{x^{-7/2}}{\sqrt{1+x^5}}, x\right]$$

$$-\frac{2\sqrt{1+x^5}}{5\, x^{5/2}}$$

$$\text{Int}\left[\frac{x^{3/2}}{\sqrt{1+x^5}}, x\right]$$

$$\frac{2}{5}\,\text{ArcSinh}\left[x^{5/2}\right]$$

$$\text{Int}\left[\frac{x^{13/2}}{\sqrt{1+x^5}}, x\right]$$

$$\frac{1}{5}\, x^{5/2}\sqrt{1+x^5} - \frac{1}{5}\,\text{ArcSinh}\left[x^{5/2}\right]$$

- *Mathematica* is unable to integrate the expression when n is positive:

$$\int \frac{x^{-7/2}}{\sqrt{1+x^5}}\, dx$$

$$-\frac{2\sqrt{1+x^5}}{5\, x^{5/2}}$$

$$\int \frac{x^{3/2}}{\sqrt{1+x^5}}\, dx$$

$$\int \frac{x^{3/2}}{\sqrt{1+x^5}}\, dx$$

$$\int \frac{x^{13/2}}{\sqrt{1+x^5}}\, dx$$

$$\int \frac{x^{13/2}}{\sqrt{1+x^5}}\, dx$$

Maple is able to integrate the expression for n mod 10 = 3, but the result for n=13 is incorrect:

```
int (x^(-7/2)/sqrt(1+x^5), x);
```

$$-\frac{2\,(1+x)\,\left(1-x+x^2-x^3+x^4\right)}{5\,x^{5/2}\,\sqrt{1+x^5}}$$

```
int (x^(3/2)/sqrt(1+x^5), x);
```

$$\frac{2}{5}\,\mathrm{ArcSinh}\left[x^{5/2}\right]$$

```
int (x^(13/2)/sqrt(1+x^5), x);
```

$$\frac{x^3\,\left(1+x^5\right)}{5\,\sqrt{x\,\left(1+x^5\right)}}-\frac{1}{5}\,\mathrm{ArcSinh}\left[x^{5/2}\right]$$

Algebraic Function Integration Problem 16

$$\int\sqrt{a+\frac{b}{x}}\,dx$$

- The *Rubi* result is a simple 2 term sum involving the hyperbolic arctangent:

```
Int[Sqrt[a + b/x], x]
```

$$\sqrt{a+\frac{b}{x}}\;x+\frac{b\,\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{\sqrt{a}}$$

- The *Mathematica* result is a more complicated 2 term sum involving the logarithm:

$$\int\sqrt{a+\frac{b}{x}}\,dx$$

$$\sqrt{a+\frac{b}{x}}\;x+\frac{b\,\text{Log}\left[b+2\,a\,x+2\,\sqrt{a}\,\sqrt{a+\frac{b}{x}}\;x\right]}{2\,\sqrt{a}}$$

- The *Maple* result is a more complicated 2 term sum involving 2 logarithms:

```
int(sqrt(a+b/x),x);
```

$$\sqrt{a+\frac{b}{x}}\;x+\frac{b\,\sqrt{a+\frac{b}{x}}\;x\left(-\text{Log}[2]+\text{Log}\left[\frac{b+2\,a\,x+2\,\sqrt{a}\,\sqrt{b\,x+a\,x^2}}{\sqrt{a}}\right]\right)}{2\,\sqrt{a}\,\sqrt{b\,x+a\,x^2}}$$

Algebraic Function Integration Problem 17

$$\int \sqrt{-a + \frac{b}{x}}\, dx$$

- The *Rubi* result is a simple 2 term sum free of the imaginary unit:

```
Int[Sqrt[-a + b/x], x]
```

$$\sqrt{-a + \frac{b}{x}}\, x - \frac{b\, \text{ArcTan}\left[\frac{\sqrt{-a+\frac{b}{x}}}{\sqrt{a}}\right]}{\sqrt{a}}$$

- The *Mathematica* result is a 2 term sum involving the imaginary unit:

$$\int \sqrt{-a + \frac{b}{x}}\, dx$$

$$\sqrt{-a + \frac{b}{x}}\, x + \frac{i\, b\, \text{Log}\left[2 \sqrt{-a + \frac{b}{x}}\, x + \frac{i\,(b-2\,a\,x)}{\sqrt{a}}\right]}{2\sqrt{a}}$$

- The *Maple* result is a more complicated 2 term sum free of the imaginary unit:

```
int(sqrt(-a + b / x), x);
```

$$\sqrt{-a + \frac{b}{x}}\, x + \frac{b \sqrt{-a + \frac{b}{x}}\, x\, \text{ArcTan}\left[\frac{-b+2\,a\,x}{2\sqrt{a}\,\sqrt{b\,x-a\,x^2}}\right]}{2\sqrt{a}\,\sqrt{b\,x-a\,x^2}}$$

Algebraic Function Integration Problem 18

$$\int \frac{1}{x+\sqrt{-2+3x-x^2}}\,dx$$

- The *Rubi* result is a relatively simple 4 term sum free of the imaginary unit:

```
Simplify[Int[1/(x + Sqrt[-2 + 3 x - x^2]), x]]
```

$$\text{ArcTan}\left[\frac{\sqrt{-2+3x-x^2}}{1-x}\right]+\frac{3\,\text{ArcTan}\left[\frac{-1+x+2\sqrt{-2+3x-x^2}}{\sqrt{7}\,(-1+x)}\right]}{\sqrt{7}}-\frac{1}{2}\,\text{Log}\left[\frac{1}{-1+x}\right]+\frac{1}{2}\,\text{Log}\left[\frac{x+\sqrt{-2+3x-x^2}}{-1+x}\right]$$

- The *Mathematica* result is huge and involves the imaginary unit:

```
Simplify[∫ 1/(x + Sqrt[-2 + 3 x - x^2]) dx]
```

$$
\begin{aligned}
&\frac{1}{56}\Bigg(-28\,\mathrm{ArcSin}[3-2x]+12\sqrt{7}\,\mathrm{ArcTan}\Big[\frac{-3+4x}{\sqrt{7}}\Big]+\\
&2\sqrt{14-42\,i\sqrt{7}}\;\mathrm{ArcTan}\Big[\Big(7\Big(58-150\,i\sqrt{7}+45\,i\left(7\,i+13\sqrt{7}\right)x+\\
&\qquad\left(807-831\,i\sqrt{7}\right)x^2+\left(-768+504\,i\sqrt{7}\right)x^3+4\left(59-27\,i\sqrt{7}\right)x^4\Big)\Big)\Big/\\
&\quad\Big(12\left(21\,i+29\sqrt{7}\right)x^4+x\left(4683\,i-1719\sqrt{7}-416\sqrt{14-42\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\right)-\\
&\qquad 8x^3\left(-21\,i+216\sqrt{7}+16\sqrt{14-42\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\right)+\\
&\qquad 6\left(-315\,i+43\sqrt{7}+32\sqrt{14-42\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\right)+\\
&\qquad 3x^2\left(-1071\,i+945\sqrt{7}+128\sqrt{14-42\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\right)\Big)\Big]-\\
&\frac{1}{\sqrt{\frac{1}{14}\left(1+3\,i\sqrt{7}\right)}}\,2\,i\left(-i+3\sqrt{7}\right)\mathrm{ArcTan}\Big[\Big(7\Big(58+150\,i\sqrt{7}+\left(-315-585\,i\sqrt{7}\right)x+\\
&\qquad\left(807+831\,i\sqrt{7}\right)x^2+\left(-768-504\,i\sqrt{7}\right)x^3+4\left(59+27\,i\sqrt{7}\right)x^4\Big)\Big)\Big/\\
&\quad\Big(\left(252\,i-348\sqrt{7}\right)x^4+8x^3\left(21\,i+216\sqrt{7}+16\sqrt{14+42\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\right)-\\
&\qquad 6\left(315\,i+43\sqrt{7}+32\sqrt{14+42\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\right)-3x^2\Big(1071\,i+945\sqrt{7}+\\
&\qquad 128\sqrt{14+42\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\Big)+x\left(4683\,i+1719\sqrt{7}+416\sqrt{14+42\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\right)\Big)\Big]+\\
&14\,\mathrm{Log}\left[2-3x+2x^2\right]-\frac{\left(-i+3\sqrt{7}\right)\mathrm{Log}\left[64\left(2-3x+2x^2\right)^2\right]}{\sqrt{\frac{1}{14}\left(1+3\,i\sqrt{7}\right)}}-\frac{\left(i+3\sqrt{7}\right)\mathrm{Log}\left[64\left(2-3x+2x^2\right)^2\right]}{\sqrt{\frac{1}{14}\left(1-3\,i\sqrt{7}\right)}}+\\
&\frac{1}{\sqrt{\frac{1}{14}\left(1-3\,i\sqrt{7}\right)}}\\
&\left(i+3\sqrt{7}\right)\mathrm{Log}\Big[\left(2-3x+2x^2\right)\Big(12\left(-2\,i+\sqrt{7}\right)+2\left(-7\,i+3\sqrt{7}\right)x^2+\\
&\qquad 7\,i\sqrt{2-6\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}-6x\left(-6\,i+3\sqrt{7}+i\sqrt{2-6\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\right)\Big)\Big]+\\
&\frac{1}{\sqrt{\frac{1}{14}\left(1+3\,i\sqrt{7}\right)}}\left(-i+3\sqrt{7}\right)\mathrm{Log}\Big[\left(2-3x+2x^2\right)\Big(12\left(2\,i+\sqrt{7}\right)+2\left(7\,i+3\sqrt{7}\right)x^2-\\
&\qquad 7\,i\sqrt{2+6\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}-6x\left(6\,i+3\sqrt{7}-i\sqrt{2+6\,i\sqrt{7}}\,\sqrt{-2+3x-x^2}\right)\Big)\Big]\Bigg)
\end{aligned}
$$

- The *Maple* result is a complicated 7 term sum:

```
simplify(int(1/(x + sqrt(-2 + 3*x - x^2)), x));
```

$$-\frac{1}{2}\text{ArcSin}[3-2x]+\frac{3\,\text{ArcTan}\left[\frac{-3+4x}{\sqrt{7}}\right]}{2\sqrt{7}}-\frac{6\sqrt{-\frac{2-3x+x^2}{(4-3x)^2}}\,\text{ArcTan}\left[\sqrt{7}\sqrt{-\frac{2-3x+x^2}{(4-3x)^2}}\right]}{\sqrt{7}\sqrt{-2+3x-x^2}}+$$

$$\frac{9x\sqrt{-\frac{2-3x+x^2}{(-4+3x)^2}}\,\text{ArcTan}\left[\sqrt{7}\sqrt{-\frac{2-3x+x^2}{(-4+3x)^2}}\right]}{\sqrt{14}\sqrt{-4+6x-2x^2}}-\frac{2\sqrt{-\frac{2-3x+x^2}{(4-3x)^2}}\,\text{ArcTanh}\left[\frac{x}{(-4+3x)\sqrt{-\frac{2-3x+x^2}{(4-3x)^2}}}\right]}{\sqrt{-2+3x-x^2}}+$$

$$\frac{3x\sqrt{-\frac{2-3x+x^2}{(4-3x)^2}}\,\text{ArcTanh}\left[\frac{x}{(-4+3x)\sqrt{-\frac{2-3x+x^2}{(4-3x)^2}}}\right]}{2\sqrt{-2+3x-x^2}}+\frac{1}{4}\text{Log}\left[2-3x+2x^2\right]$$